Quantum computing of fluid dynamics using the hydrodynamic Schrödinger equation

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Simulating fluid dynamics on a quantum computer[1] is intrinsically difficult due to the nonlinear and non-Hamiltonian nature of the Navier–Stokes equation (NSE)[2]. We propose a framework for quantum computing of fluid dynamics based on the hydrodynamic Schrödinger equation (HSE), which can be promising in simulating three-dimensional turbulent flows in various engineering applications[3]. The HSE is derived by generalizing the Madelung transform to compressible/incompressible flows with finite vorticity and dissipation. Since the HSE is expressed as a unitary operator on a two-component wave function, it is more suitable than the NSE for quantum computing. The flow governed by the HSE can resemble a turbulent flow consisting of tangled vortex tubes (see figure 1) with the five-thirds scaling of energy spectrum. We develop a prediction-correction quantum algorithm to solve the HSE. This algorithm is implemented for simple flows on the quantum simulator Qiskit with exponential speedup.

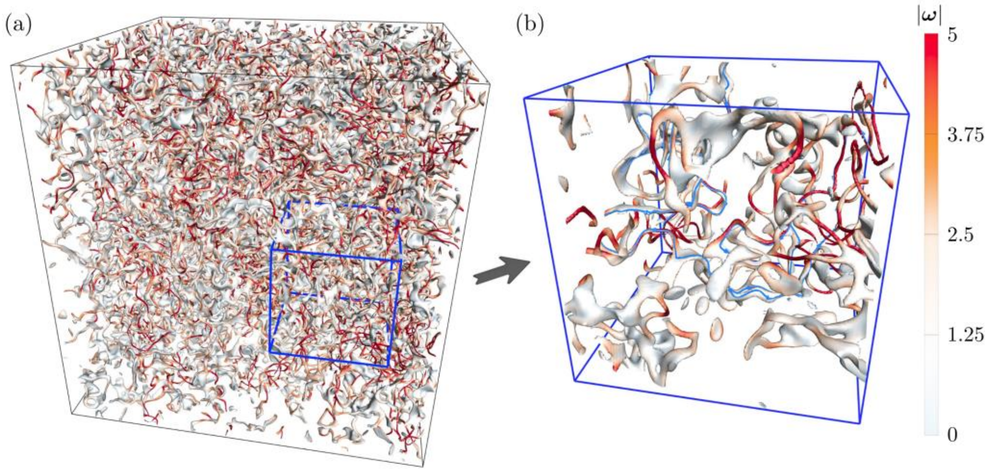

Figure 1: Visualization of the tangled vortex tubes for the decaying HIT in the ISF. (a) Isosurface of $s_1 s_2 s_3 = 0.18$ color-coded by $|\boldsymbol{\omega}|$. (b) Close-up view of the region marked by the blue box in (a). Some vortex lines (blue) are integrated and plotted on the isosurface in (b).

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[2] Liu et al., *Proc. Natl. Acad. Sci. U. S. A.* **118**, e2026805118 (2021).

[3] Meng and Yang, arXiv: 2302.09741 (2023).