Skin friction law in the compressible turbulent channel flows

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In the last one or two decades, researchers studied the skin friction law based on the energy dissipation function[1]. Abe and Antonia [2] found the skin friction law in incompressible wall-bounded flows followed a logarithmic law, i.e. for channel flows $u_b^+ = \sqrt{2/C_f} = 2.54ln(h^+) + 2.41$, for pipe flows $u_b^+ = 2.54ln(R^+) + 1.31$.

Our work is to extend the former one to the compressible case, i.e. to establish a formula for the friction coefficient in compressible turbulent channel flows. It is found that u_b can be decomposed into the compressible part u_{com} and incompressible part u_{in} as

$$u^+_{com} = \frac{1}{h}\int_0^h \frac{\overline{\rho}}{\rho_b}(u^+ - u^+_{TL})dy, \quad (1)$$

$$u^+_{in} = \int_0^{h^*} \overline{\mu}^+ \frac{\partial \overline{u}^+}{\partial y^+}\frac{u^+_{TL}}{\partial y^*}dy^* + \int_0^{h^*} \overline{(-\rho u''v'')}^+ \frac{u^+_{TL}}{\partial y^*}dy^*. \quad (2)$$

Here, u_b is the bulk velocity, ρ_b is bulk-averaged density, u_{TL} is the velocity after TL transformation, y^* is semi-local scale, $\overline{(-\rho u''v'')}$ is the Reynolds stress and the superscript $^+$ means a nondimensionalization by the classical wall units. It should be noted that $\overline{\mu}^+ \frac{\partial \overline{u}^+}{\partial y^+}$, $\frac{u^+_{TL}}{\partial y^*}$ and $\overline{(-\rho u''v'')}^+$ are Mach-independent variables, based on Morkovin's scaling and the stress balance. Therefore, u^+_{in} is also a Mach-independent variable, and it can be modelled with incompressible conclusions.

For the compressible part u^+_{com}, which is a function of Reynolds and Mach number, we arrived at the following formula as

$$u^+_{com} = f(Re^*_\tau, Ma) \approx 0.083Ma^2 ln(Re^*_\tau) + (0.37Ma^2 + 1.21Ma) \quad (3)$$

for $Re^*_\tau > 300$ based on the profiles obtained by our recent proposed iterative method[3] with $0.3 \leq Ma \leq 4.1$ (interval 0.2) and $6000 \leq Re \leq 384000$. Using the same data, we also have the results for the incompressible part as

$$\int_0^{h^*} \overline{\mu}^+ \frac{\partial \overline{u}^+}{\partial y^+}\frac{u^+_{TL}}{\partial y^*}dy^* \approx 9.08, \int_0^{h^*} \overline{(-\rho u''v'')}^+ \frac{u^+_{TL}}{\partial y^*}dy^* \approx 2.32ln(Re^*_\tau) - 5.20 \quad (4)$$

Therefore, we get a semi-empirical relationship for u_b^+, and thus the skin-friction law for compressible turbulent channel flows (with ρ_w/ρ_b).

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[1] Laadhari F, *Phys. Fluids.* **19**, 038101 (2007).

[2] Abe H, Antonia R A, *J. Fluid Mech.* **798**,140-164 (2016).

[3] Song Y, Zhang P, Xia Z. *Phys. Rev. Fluids* accepted in press, (2023).