## New Exact Betchov-like Relation for the Helicity Flux in Homogeneous Turbulence<sup>\*</sup>

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In homogeneous and isotropic turbulence, the relative contributions of different physical mechanisms to the energy cascade can be quantified by an exact decomposition of the energy flux <sup>1 2</sup>  $\langle \Pi^{H,\ell} \rangle$ . We extend the formalism to the transfer of kinetic helicity across scales, important in the presence of large-scale mirror breaking mechanisms, to identify physical processes resulting in helicity transfer and quantify their contributions to the mean flux in the inertial range. All subfluxes transfer helicity from large to small scales. About 50% of the mean flux is due to the scale-local vortex flattening  $\langle \Pi^{H,\ell}_{s,SS} \rangle$  and vortex twisting  $\langle \Pi^{H,\ell}_{s,S\Omega} \rangle$ . We derive a new exact relation between these effects, similar to the Betchov relation for the energy flux, revealing that the mean contribution of the former is three times larger than that of the latter, see fig. 1. Multi-scale effects account for the remaining 50% of the mean flux, with approximate equipartition between multi-scale vortex flattening  $\langle \Pi^{H,\ell}_{m,SS} \rangle$ , twisting  $\langle \Pi^{H,\ell}_{m,S\Omega} \rangle$  and entangling  $\langle \Pi^{H,\ell}_{m,\Omega\Omega} \rangle$ .



Figure 1: Decomposition of the helicity flux across the scales normalized by the mean helicity dissipation rate  $\varepsilon_H$  as a function of the wavenumber  $k = \pi/\ell$  times the Kolmogorov microscale  $\eta$ .

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