Emergence of Order in 2D Euler Turbulence, an Isolated System

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Euler flow is an isolated system because of an absence of viscosity and external forcing. It has been shown that three-dimensional Euler turbulence evolves from order to disorder[1], as is expected for an isolated energy-conserving system. In contrast, the asymptotic states of two-dimensional (2D) Euler turbulence exhibit large-scale flow structures due to nonzero energy transfers among small wavenumber modes, as we[2] show using numerical simulations. Figure 1(A) illustrates the initial and asymptotic states of the three runs performed by us[2]. These asymptotic states, which depend on the initial conditions, are out of equilibrium, and they differ from the predictions of equilibrium states by Onsager[3] and Kraichnan[4].

We observe that the asymptotic states of the runs are more ordered than the initial states. To quantify the order in the flow, we propose "hydrodynamic entropy", $-\sum_{\mathbf{k}} p_{\mathbf{k}} \log(p_{\mathbf{k}})$ where $p_{\mathbf{k}} = E(\mathbf{k})/E$. The above formula is motivated by Shannon's famous formula. We show that this entropy decreases with time for a significant duration, even though the system is isolated with no dissipation and no contact with a heat bath (see Fig. 1(B)).

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[1] Cichowlas et al., *Phys. Rev. Lett.* **95**, 264502 (2005).
[2] Verma and Chatterjee, 7, 114608 (2022).
[3] Onsager, Il Nuovo Cimento **6**, 279 (1949).
[4] Kraichnan, J. Fluid Mech. **59**, 745 (1973).

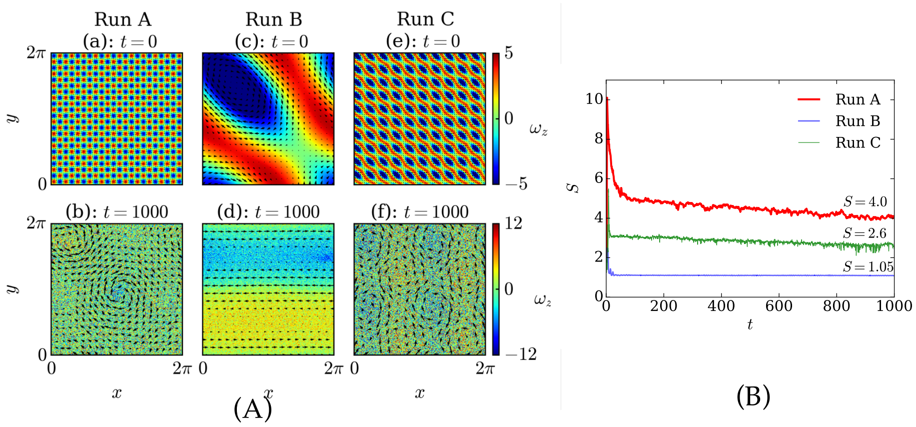

Figure 1: (A) For runs A–C of 2D Euler turbulence: (a), (c), (e) the initial states. (b), (d), (f) the final states, which are more ordered than the initial states. (B) Temporal evolution of hydrodynamic entropies for the three runs that shows decrease in entropy with time.