Point-vortex statistical mechanics applied to turbulence without vortex stretching

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In two-dimensional (2D) turbulence, energy shows a tendency to accumulate at large scales. Conversely, in three-dimensional (3D) turbulence, energy cascades towards small scales. Vortex stretching plays an important role in this distinction between 2D and 3D turbulence. Our aim is to understand the effect of vortex stretching by removing this term from the Navier-Stokes equations[1][2]. The equation for the vorticity $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$ is in that case,

$$\partial_t \boldsymbol{\omega} + \boldsymbol{u} \cdot \nabla \boldsymbol{\omega} = -\boldsymbol{\nabla} P_\omega + \boldsymbol{D}_\omega, \tag{1}$$

with $\boldsymbol{\nabla} P_\omega$ a pressure gradient term ensuring incompressibility and $\boldsymbol{D}_\omega$ (hyper)viscous damping. We observe that in 3D turbulence from which vortex-stretching is removed, such condensation is observed, associated with an inverse cascade of helicity[3].

We generalize a 2D point-vortex statistical mechanics approach to our 3D turbulence without vortex stretching to investigate the nature of the condensate. Our analysis reveals that the large-scale structure of this condensate is characterized by a hyperbolic sine relation between vorticity and velocity, analogous to the sinh relation between vorticity and stream function observed in freely decaying 2D turbulence in periodic domains.

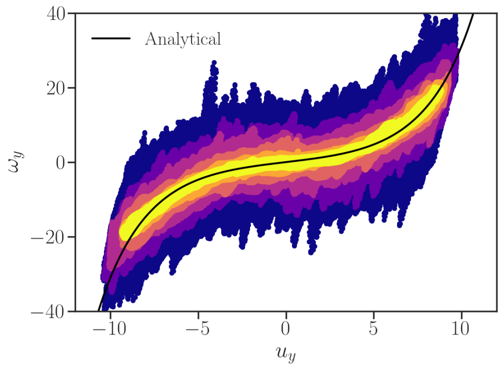

Figure 1: Hyperbolic sine relation between components of the vorticity and velocity in a direct numerical simulation during the freely decaying phase (from dark to light) of turbulence without vortex-stretching.

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[1] Bos, *J. Fluid Mech.* **915**, A121 (2021).

[2] Wu and Bos, *J. Fluid Mech.* **929** (2021).

[3] Wu and Bos, *Phys. Rev. Fluids* **7(9)**, 094601 (2022).