Signature of pressure-balanced relaxed states in binary fluid turbulence

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If the forcing of a fully developed turbulent flow is abruptly quenched, the system tends to self-organize itself through the process of turbulent relaxation. Such a relaxation is believed to lead to Beltrami-Taylor (BT) alignment of the current ($\boldsymbol{j}$) and the magnetic field ($\boldsymbol{b}$) in cosmic plasmas. BT states can be obtained by the theory of selective decay. However, this theory cannot explain pressure-gradient balanced (PB) relaxed states which is often observed numerically in incompressible Hydrodynamics (HD) and magnetohydrodynamics (MHD). Recently, we propose a new principle of vanishing nonlinear transfer (PVNLT) to study turbulent relaxation[1]. According to PVNLT, a turbulent state relaxes toward a state where the average nonlinear scale-to-scale transfer rates of all the inviscid invariants identically vanishes within the inertial range. This theory successfully explains the PB states and recovers the BT states under the limit of low plasma-β in both HD and MHD.

In the current study, we numerically obtain such PB states in the turbulent relaxation of a binary mixture of two incompressible fluids *e.g.*, oil and water to complex fluids such as active fluids. Study of binary fluids (BF) has relevance in bubbly flows, alloys, atmospheric studies etc. Evolution of BF is governed by the coupled dynamics of Cahn-Hilliard-Navier-Stokes equation[2]. In addition to the two known invariants total energy $E\,(=\int(u^2+\xi Q^2)/2\ d\tau)$ and scalar energy $S\,(=\int(\phi^2/2)\ d\tau)$, we found that potential enstrophy $\Omega\,(=\int(\boldsymbol{\omega}\cdot\boldsymbol{Q})^2\ d\tau)$ is also a non-trivial inviscid invariant for this system, where $\boldsymbol{u}$ is the velocity field, ϕ is the order parameter, $\boldsymbol{Q}=\boldsymbol{\nabla}\phi$, $\boldsymbol{\omega}=\boldsymbol{\nabla}\times\boldsymbol{u}$ and ξ is the activity parameter[3]. Following relaxed states are obtained for BF by constructing the two-point correlators of the above invariants in physical space and setting the corresponding nonlinear transfer rate to zero-

$$\boldsymbol{u}\times\boldsymbol{\omega}-\xi\boldsymbol{Q}(\boldsymbol{\nabla}\cdot\boldsymbol{Q})-\boldsymbol{\nabla}P=\boldsymbol{\nabla}\Phi, \tag{1}$$

$$\boldsymbol{u}\cdot\boldsymbol{Q}=0 \text{ and } \boldsymbol{u}\cdot\boldsymbol{\nabla}(\boldsymbol{\omega}\cdot\boldsymbol{Q})=0, \tag{2}$$

where Φ is an arbitrary scalar field. We perform DNS of the BF system with 256^3 grid points below the critical temperature upto a statistical steady state. At 31.6 nonlinear times (t_{nl}), the forcing is quenched and the relaxed states are analysed by plotting the average values of different nonlinear terms with time. We observe that, inside the single fluid bulk ($|\boldsymbol{Q}|\sim 0$), $\boldsymbol{u}\times\boldsymbol{\omega}$ and $\boldsymbol{\nabla}P$ balance each other and $\boldsymbol{\nabla}P$ is balanced by $-\xi\boldsymbol{Q}\,(\boldsymbol{\nabla}\cdot\boldsymbol{Q})$ on the two-fluid interface ($|\boldsymbol{Q}| >> 1$) with $\boldsymbol{u}$ becoming zero eventually, which satisfies (1) and (2) (Fig. 1).

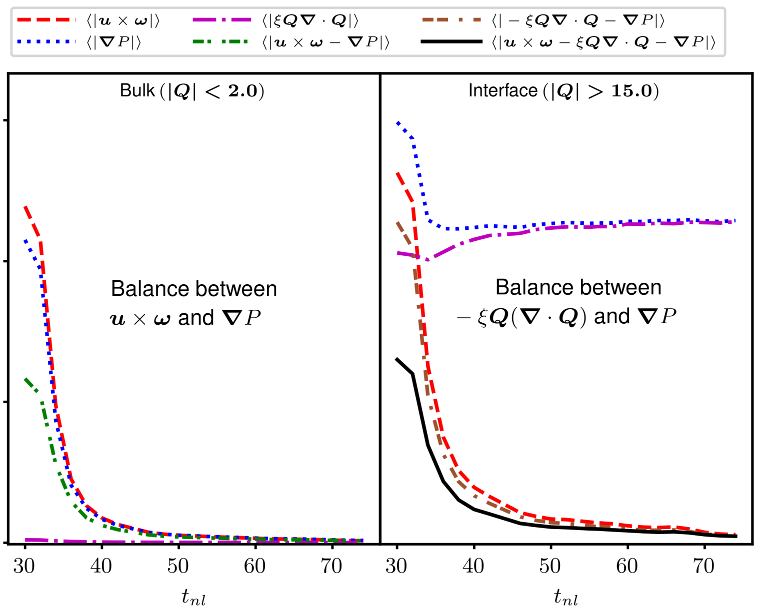

Figure 1: Average of absolute values of different nonlinear terms with time.

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[1]Banerjee, Halder and Pan, arXiv:2209.12735 (2022)(Accepted in *Phys. Rev. E Lett.*)

[2]Pan and Banerjee, *Phys. Rev. E* **106**, 025104 (2022)

[3]Pan, Banerjee and Halder, *In preparation*, (2023)