Mechanisms for three-dimensional instabilities in the wake behind a cylinder

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The two-dimensional time-periodic viscous fluid flow (the von Kármán vortex street) behind a circular cylinder is linearly unstable to two modes of three-dimensional perturbations, labelled A and B, at critical Reynolds numbers $\mathrm{Re}_A \approx 190$ and $\mathrm{Re}_B \approx 260$ and critical wavelengths $\lambda_A \approx 4$ and $\lambda_B \approx 0.8$ diameters of the cylinder respectively[1,2]. Despite previous research narrowing down the source of these instabilities to the vortex formation region[3,4], identifying the underlying physical mechanisms remains a challenging problem due to the complexity of the non-stationary flow in this region. In this talk, we address the problem by leveraging the fact that the essential mechanisms for modes A and B are preserved in the limits of infinitely large and short wavelengths, respectively, and by exploiting the asymptotic expansion of the solution to simplify the problem.

For mode A, we show that the three-dimensional flow tends to organise such that, in each streamwise slice, it still corresponds to the base-flow two-dimensional solution, but at shifted times[5]. This suggests an explanation for the observed pattern of the perturbations and provides a possible criterion for the unambiguous classification of three-dimensional instabilities of this kind.

For mode B, we propose a way of justifying the hypothesis[4,6,7] that local instability along a closed trajectory (the so-called orbit 3) in the vortex formation region is responsible for the instability[8]. This hypothesis was initially built upon a WKBJ-based analysis in the limit of inviscid perturbations with infinite wavenumber. The lack of justification for these assumptions resulted in conflicting views in the literature[6,7]. Our approach justifies the infinite-wavenumber assumption and shows new evidence that the "inviscid" instability along orbit 3 is, indeed, at the core of the mode B instability.

Finally, we discuss the selection of critical parameters for the instabilities and rationalise the non-trivial dependence of the growth rate on the spanwise wavenumber by identifying the contribution of basic physical mechanisms to flow destabilisation. Our results have broad implications for a general understanding of the underlying mechanisms for large- and short-wavelength three-dimensional instabilities.

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