Statistical field theory for a stochastic linear advection-stretching model for turbulence

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A major obstacle in developing a statistical field theory of turbulence is the analysis of the functional equations that govern the complete statistics of the flow field.[1] Simplified models of turbulence may help to develop such a statistical framework. In this contribution, we discuss the stochastic linear advection and stretching of an incompressible passive vector field as a model for small-scale turbulence.[2,3] The model encompasses non-Gaussian statistics due to an intermittent transfer of energy from large scales to small scales (see Figure 1), thereby displaying hallmark features of turbulence. We explore this model using the Hopf functional formalism, which naturally leads to a decomposition of the complex non-Gaussian statistics into Gaussian sub-ensembles based on different realizations of advection and stretching.

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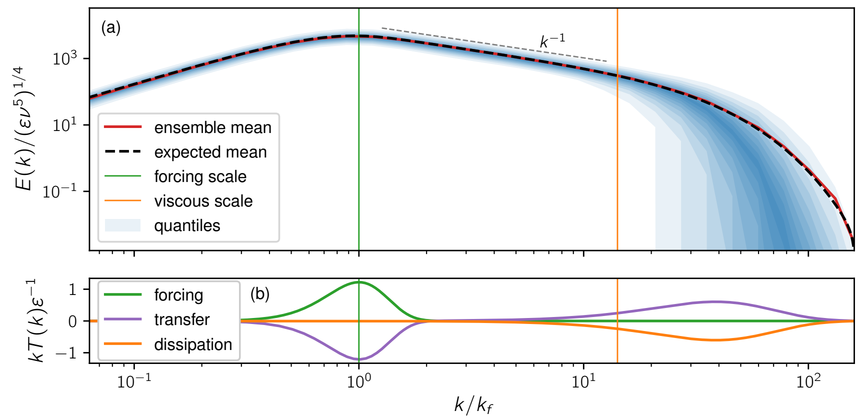

Figure 1: (a) Energy spectrum of the model computed as ensemble mean from numerical simulations (red) and from theory (black, dashed), normalized by energy injection rate ε and viscosity ν. The blue, shaded area marks the quantiles of the energy spectrum across the different realizations of stretching, showing increasingly strong fluctuations at small scales. (b) Mean spectral kinetic energy balance across wavenumbers k, comparing the terms $T(k)$ of the energy injection due to the forcing, the energy transfer from large to small scales, and the viscous dissipation, all nondimensionalized by k and ε.

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