The vKHM Relation for Energy Transfer in Isothermal Compressible Turbulence

Alexei G. Kritsuk*

Statistics of three-dimensional isothermal compressible turbulence are analyzed in an asymptotic limit of large Reynolds numbers. Based on the inviscid invariance of total energy, a number of compressible analogues of the von Kármán-Howarth-Monin (vKHM) equation, describing the scale-by-scale energy budget, have been derived assuming homogeneity.[1,2,3,4,5] We introduce a set of selection criteria to single out a unique exact relation, characterizing the structure of inertial range turbulence, and illustrate these with high-resolution direct numerical simulations (DNS) of near-isothermal turbulence at Mach numbers M_t = 0.28, 0.60, 1.25, and 1.90. These numerical experiments probe turbulence statistics in a wide range of compressibility regimes: nearly incompressible, weakly compressible, transonic, and supersonic (see Fig. 1a for the probability density functions (PDFs) of density fluctuations). We use the DNS to show how the nonlinear vKHM equation captures the growing role of density fluctuations and effects of pressure dilatation on the energy cascade as the turbulence turns from incompressible to supersonic (see Fig 1b for a transonic example).

The simulations employed high-order accurate nonlinear filter methods with adaptive dissipation control[6] and utilized an LRAC allocation AST21004 and allocation MCA07S014 of the ACCESS program. A.K. was supported in part by the NASA Grant 80NSSC22K0724.

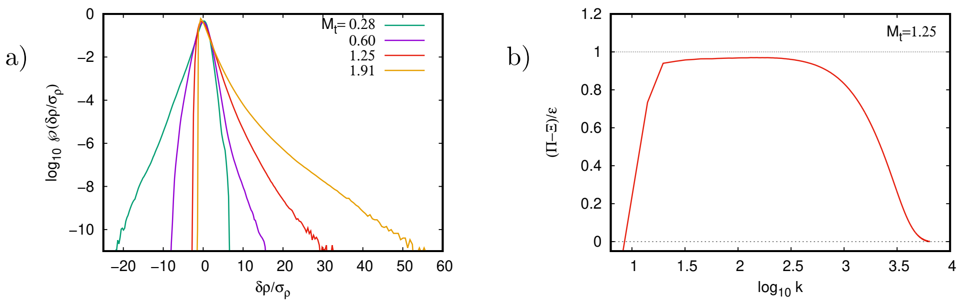

Figure 1: Sample results from the four 2048^3 turbulence DNS: (a) Mach number dependence for the PDF of density fluctuations. (b) Kinetic energy flux, $\Pi(k)$, corrected for a distributed source associated with pressure-dilatation effects, $\Xi(k)$, and normalized with the energy injection rate, ε, for a coarse DNS at $M_\mathrm{t} = 1.25$ with an effective $R_\lambda = 1109$.

*University of California, San Diego, La Jolla, CA 92093-0424, USA

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