Do marginally-stable manifolds chart chaos?

A. Ferraro,* G. P. Chini †, T. M. Schneider *

A new mathematical formalism that exploits the property of quasi-linear systems to self-tune towards marginally stable states is investigated and applied to the 2D strongly stratified flow problem. The asymptotic analysis of the Boussinesq equations in the limit of strong stratification naturally yields a multiscale reduced model where small-scale instabilities evolve linearly about a large-scale hydrostatic field and modify it via a nonlinear feedback term.[1] A recently introduced mathematical formalism for the integration of slow-fast QL systems exploits the tendency of these systems to self-organize about a marginal stability manifold and slaves the amplitude of the (marginal) fluctuations to the slowly-evolving mean field.[1,2]
An interesting feature of this reduced system is the two-ways coupling between the slow and the fast dynamics: the feedback produced by the fluctuations on the mean variable is not sign-definite and its effect may be stabilizing or destabilizing in nature. Here, we address two important extensions to this formalism. The first extension accommodates large-amplitude bursting events, in which temporal scale separation is transiently lost, requiring the co-evolution of the slow and the fast fields on the same temporal scale until marginal stability is re-established. The second extension yields a slow evolution equation for the wavenumber of the fastest growing mode, whose amplitude is then slaved to the mean field dynamics in condition of marginal stability to maintain a zero growth rate. Together, these extensions enable scale-selective adaptivity in both space and time. Our formalism is consistent with the idea that shear flow turbulence tracks low-dimensional state space structures (marginally stable manifolds) during slow evolutionary phases punctuated by intermittent bursting events.

*Emergent Complexity in Physical Systems lab (ECPS), EPFL Lausanne, Switzerland
†Department of Mechanical Engineering, University of New Hampshire, Durham, NH 03824

[1] Chini et al., *J. Fluid Mech.* **933**, A22 (2022).
[2] Michel and Chini, *Proc. R. Soc. A.* **475** (2019).