Bolgiano-Obukhov Scaling in Two-Dimensional Rayleigh-Bénard Convection

Roshan Samuel*and Mahendra K. Verma†

Understanding the nature of energy transfers in turbulent Rayleigh-Bénard Convection (RBC) remains a challenge. The predominant question is whether its energy cascade obeys Kolmogorov-Obukhov (KO) scaling, $E_u(k) \sim k^{-5/3}$, or Bolgiano-Obukhov (BO) scaling, $E_u(k) \sim k^{-11/5}$. Although earlier theories [1] argued for BO scaling in RBC based on observations drawn from stably-stratified flows, recent results [2] have demonstrated that 3D RBC exhibits KO scaling. However, 2D RBC remained inadequately explored in this respect.

We use high-resolution direct numerical simulations of RBC at Rayleigh numbers 10^{11} to 10^{14} and unit Prandtl number to show that 2D turbulent convection exhibits Bolgiano-Obukhov scaling. At small wavenumbers, where buoyancy feeds energy to the velocity field, kinetic energy exhibits inverse cascade [3]. Consequently, kinetic energy spectrum scales as $k^{-11/5}$ (shown in Fig. 1(a)) and the kinetic energy flux shows $k^{-4/5}$ scaling [4] [5]. Buoyancy is weakened at large wavenumbers, and this leads to a constant enstrophy cascade and k^{-3} kinetic energy spectrum (Fig. 1(a)), similar to 2D hydrodynamic turbulence. However, the temperature fluctuation spectrum (also referred to as entropy spectrum in literature) exhibits bi-spectrum with the upper branch varying as k^{-2} (Fig. 1 (b)). We also draw a connection between the entropy flux in the dissipation range and the entropy dissipation rate in the bulk.

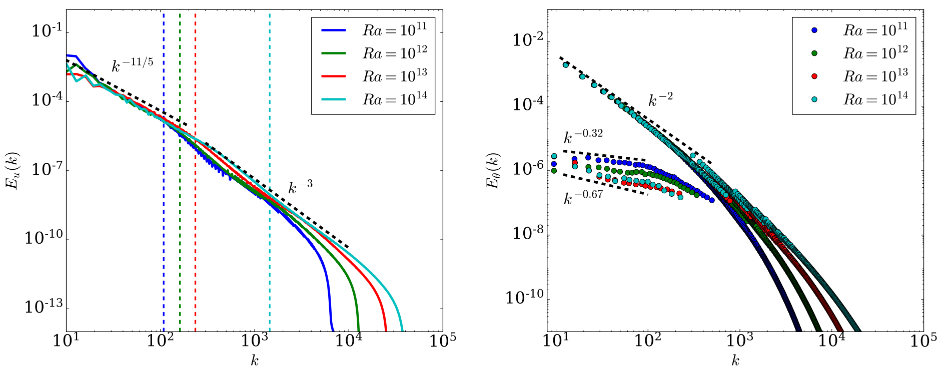

Figure 1: (a) The time-averaged kinetic energy spectra for $10^{11} \leq Ra \leq 10^{14}$ with Bolgiano wavenumber k_B marked by dashed vertical lines. (b) Corresponding entropy spectra.

*Dep. Mechanical Engineering, Indian Institute of Technology, Kanpur, UP, India
†Dep. of Physics, Indian Institute of Technology, Kanpur, UP, India
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