A lattice-Boltzmann multiple-relaxation-time model for the induction equation of magnetohydrodynamics at low magnetic Reynolds numbers

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A multiple-relaxation-time (MRT) collision model for the lattice-Boltzmann method (LBM) is devised in order to deal with the induction equations of magnetohydrodynamics. Boltzmann distributions are taken into account by means of a central-moment collision model¹. The complete induction equation within the LBM framework has been usually grounded on the most popular collision model, the single-relaxation-time $(SRT)^2$. Although the SRT approach is simple and practical, it has stability issues at larger values of magnetic diffusivity, which are relevant in the flow situations of low magnetic Reynolds numbers (R_m) . It turns out that our MRT collision model, introduced to cope with this difficulty, is able to address lattice Boltzmann simulations of comprehensive magnetohydrodynamic flow regimes. For wall-bounded flows, we furthermore establish a distance-dependent Dirichlet boundary condition (BC) for the magnetic distributions which is suitable for curved boundaries as an extension from a convection-diffusion LBM BC³. To illustrate the feasibility of the algorithm we investigate the complex dynamics of the 3D Orszag-Tang vortex problem (Figure 1.a) and the time dependence on transient flow regimes in MHD pipe flows, with different types of external non-homogenous magnetic fields (Figure 1.b).



Figure 1: (a) Absolute value of vorticity for the 3D Orszag-Tang problem with Re = 2000 and $R_m = 200$. (b) Schematics of the pipe flow setup with six covering magnetic slabs.

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