Tracking complex singularities of fluids on log-lattices

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In 1981, Frisch and Morf[1] postulated the existence of complex singularities in solutions of Navier-Stokes equations. Since then, it was mathematically confirmed in the one-dimensional Burgers equation, a 1D surrogate of the Navier-Stokes equation, where complex singularities collapse to the real axes in the inviscid limit. When a viscosity ν is added, the singularities are repelled from the real axis, the closest one being constantly at a distance $O(\nu^{3/4})$ of the real line. In 3D, progress on the conjectured existence of complex singularities is impeded by the computational burden involved in the simulation of the Euler or Navier-Stokes equations at a high Reynolds number.

We investigate this conjecture in the case of fluid dynamics on log-lattice[2], where the computational burden is logarithmic concerning ordinary fluid. We analyse properties of potential complex singularities in both 1D and 3D for lattices of different steps. Dominant complex singularities are tracked using the singularity strip method, which is based on the observation that the behaviour of the energy spectrum at large wavenumber k is dominated by the position of the singularity closest to the real axis and decays like $\exp(2\delta k)$, where δ is the imaginary part of the corresponding singularity. Fitting the large wavenumber tail of the energy spectrum as a function of time, one then gets an estimate of $\delta(t)$, and real singularity occurs when $\delta(t) = 0$. So far, studies have only identified an exponential decaying regime for $\delta(t)$, meaning no finite time blow-up by extrapolation. However, we cannot guarantee that this extrapolation is correct due to numerical limitations.

In the first part, we validate the close connection between fluid dynamics on log-lattice and real fluid dynamics by focusing on the 1D Burgers equation, where dominant complex singularities are tracked using the singularity strip method. In the second part, we extend this technique to 3D to obtain new scaling regarding the approach to the real axis and the influence of normal, hypo and hyper dissipation (Fig. 1).

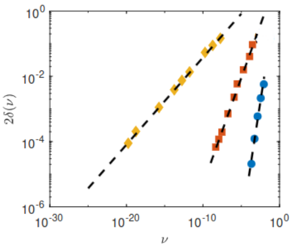

Figure 1: Width of analycity strip 2δ as a function of viscosity, for stationary dynamics for Navier-Stokes, for hypo-viscous case (blue circle), viscous case (red squares), hyper-viscous case (yellow diamond).

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[1] Frisch and Morf, *Phys. Rev.* **23**, 5, 2673-2705 (1981).
[2] Campolina and Mailybaev, *Nonlinearity*, **34**, 7, 4684, (2021)