Closure modelling of inertial particle-pair behaviour using the kinetic PDF approach

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Two-fluid modelling of dispersed particle-laden flows is applicable to a range of industrial and environmental applications, however the development of models that are able to capture the many and varied effects that turbulence can induce on the particle phase remains a challenge. Such effects include but are not limited to clustering, enhancement and suppression of settling velocity, and turbulent mixing of particles[1]. These phenomena are intrinsically related to the spatial structures that are present in the underlying turbulent flow field, and consequently a modelling approach which incorporates an appropriate physical description of such flow features is required in order to integrate these behaviours into present two-fluid models.

A suitable framework for such model development is provided by the kinetic probability density function (PDF) approach, which through statistical treatment of the flow field naturally includes the detail of the flow correlation structures within the model description[2]. Previously, it has been demonstrated that the kinetic PDF approach identifies two key contributions to the particle mass flux balance in inhomogeneous turbulent flows, of which one has previously not been accounted for in existing two-fluid modelling approaches[3]. This additional contribution takes the form of a body force that emerges from inhomogeneities in the sampling of turbulence by particles, and is important in quantifying the degree of clustering exhibited in particle-pair dispersion. Whilst the existence of this contribution has to date only been shown numerically, the kinetic PDF approach also offers a tractable methodology for developing closure models for mass flux expressions that can be incorporated into two-fluid frameworks. The present work addresses this potential by applying a cumulant expansion to the unclosed terms in the kinetic PDF approach[4], and truncating at the second-order to obtain a Gaussian closure model. This permits analytical modelling of the derived expressions, which can then be closed in terms of the Lagrangian particle equation of motion and known carrier flow correlation functions. It is shown that the developed closure model is able to capture the additional inhomogeneous body force contribution to the particle mass flux balance, and performs best for $St \ll 1$, with accuracy decreasing as the particle inertia becomes larger and the higher-order cumulants play an increasingly more important role in the model. Nonetheless, the proposed closure is able to advance the accuracy of dispersed particle-laden flow modelling by including this previously unaccounted for effect of turbulence on the particle phase.

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