A generative model of the velocity gradient dynamics in turbulence based on normalizing flows

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The velocity gradient, $\boldsymbol{A} = \boldsymbol{\nabla u}$, contains comprehensive information on the small scales in turbulence. Modelling approaches aim at obtaining a low-dimensional description to provide insights into the turbulent dynamics.[1] Machine learning (ML) is promising in producing reduced-order models for the velocity gradient, thanks to its capabilities in forecasting time series.[2] Given direct numerical simulation (DNS) data, ML-based models can be used, for example, to (a priori) learn stochastic differential equations to model (a posteriori) Lagrangian velocity gradients.[3] To what extent successful a-priori training yields quantitatively faithful a-posteriori predictions remains an open question.

Here we use ML to construct velocity gradient models that match given turbulence statistics by design. We employ normalizing flow models[4] to accurately approximate the single-time velocity-gradient probability density function (PDF) through subsequent changes of variables. Based on that, we formulate a Fokker-Planck equation for the velocity-gradient PDF that yields the learned PDF by construction. The associated Langevin equation can be used to generate Lagrangian velocity gradient trajectories whose time correlations can be controlled by gauge terms that leave the single-time statistics unchanged. The resulting model quantitatively captures the velocity-gradient statistics obtained from DNS of incompressible, statistically isotropic turbulence at a high Reynolds number.

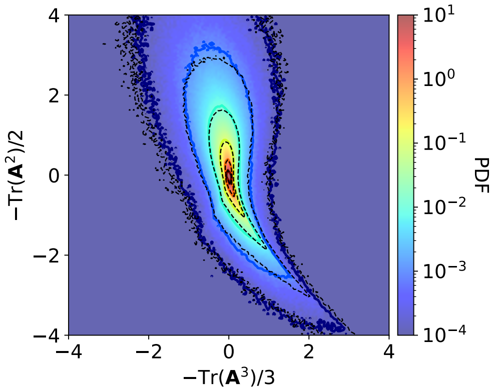

Figure 1: PDF of the velocity gradient principal invariants. The colour map and coloured contours (logarithmically equispaced) are from the DNS, while the black contours result from our ML-based model.

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