Prediction of turbulent systems from limited measurements by using data assimilation methods

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We use 4D-Var (four-dimensional variational method) and EnKF (ensemble Kalman filter) for prediction of one-dimensional Kuramoto-Sivashinsky (KS) system. We analyse the measurement conditions, such as the level of spaiotemporal sparsity and measurement noise, under which data assimilation methods can work. We find that both methods, 4D-Var and EnKF, work up to the same level of spatial sparsity. In other words, if the measurements are sparser than a particular level then both methods fail to provide accurate short-term forecasting. Moreover, we find that the condition on the level of spatial sparsity can be directly linked to the correlation dimension, which is a measure of fractal dimension occupied by the system within the phase space.

The equations for the KS system are given as,

$$u_t + uu_x + u_{xx} + \nu u_{xxxx}, \quad x \in [0, L), \tag{1}$$

where u is the system state, subscripts t and x denote partial differentiation in time and space, respectively, and ν is the viscosity parameter. Similar to the Navier–Stokes equation, the nonlinear term in the KS system is energy conserving and dissipation happens via the viscous term (νu_{xxxx}) at small scales. We numerically solve the system for $L = 32\pi$, periodic boundary conditions and $\nu = 1.0$ and 0.5 using $n = 512$ points in space and time-step of 0.1. The simulation results are considered as the true state of the system. The predictions are obtained from noisy measurements at m spatially equispaced points. Results in fig. 1 indicate that the spatial sparsity up to which data assimilation methods work may have a universal trend. The future work will be to confirm this hypothesis for fully developed turbulent systems.

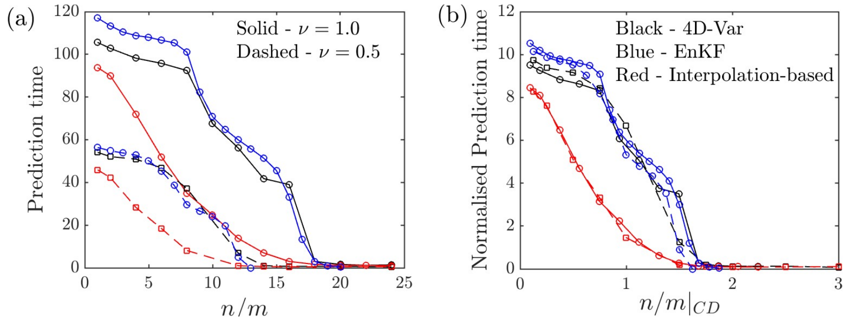

Figure 1: (a) The prediction time up to which the prediction errors remain below a threshold against the level of spatial sparsity n/m. (b) The prediction time is normalised by the inverse of the maximum Lyapunov exponent and n/m is normalised by the correlation dimension.

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