The probability density behind thermal stratifications sustained by uncertain and heterogeneous forcing

P.M. Mannix*, D. A. Ham†, J. Craske*

Thermal stratification plays a profound role in a wide range of engineering and environmental contexts, with applications including reservoir management, climate prediction, thermal comfort and energy storage. In buildings, in particular, thermal stratification has major ramifications for the transport of heat and contaminant/pathogen transport in rooms and therefore affects energy usage, health and thermal comfort. While it may be possible to discern certain features of a stratification profile for some canonical flows, heating/cooling that is heterogeneous (such as a combinations of point and distributed sources[1]) and uncertainty makes this a difficult task in the context of buildings. More generally, the underlying transport and mixing mechanisms responsible for the production, maintenance or destruction of thermal stratifications, even for canonical flows, remain poorly understood.

We address the challenge of predicting and diagnosing the physics behind thermal stratifications by embedding the problem within a probabilistic framework[2]. In doing so, we derive a Fokker-Planck-like equation for the (linear) evolution of the joint probability density associated with several key quantities, from which information about a flow's energetics[3] and thermal stratification are readily obtained as functionals. Precise information about the spatial dependence of a flow is sacrificed for detailed probabilistic information that evolves in a 'larger' space. The projection of the full functional differential equation for the system's joint probability density produces conditional expectations relating to viscous dissipation and diapycnal mixing that require closure, which we address by synthesising existing models with direct simulation data from several canonical flow configurations.

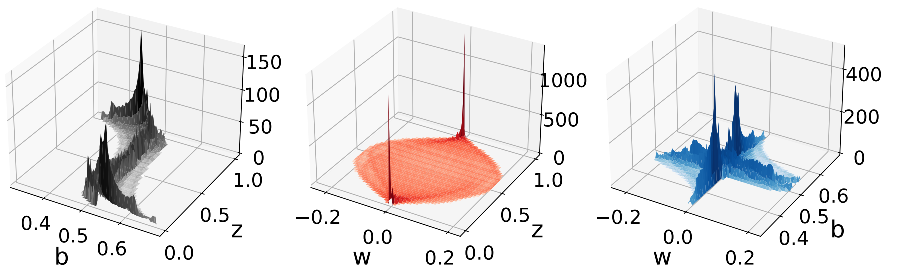

Figure 1: Marginal distributions of the joint PDF $f_{W,B,Z}(w,b,z)$ for the vertical velocity W, buoyancy B and height Z from 2D turbulent Rayleigh-Bénard convection ($Ra = 10^7$). Irreversible mixing drives B to its mean away from the boundaries (left) at which point W assumes its largest variance (middle) and is correlated with the mean buoyancy (right).

*Dep. Civil and Environmental Engineering, Imperial College London, SW7 2AZ, UK
†Dep. Mathematics, Imperial College London, SW7 2AZ, UK

[1] Mader, J. et. al., *Phys. Rev. Fluids* **6**, 023503 (2021).
[2] Pope, S.B, *Turbulent Flows*, Cambridge (2000).
[3] Tseng, Y. & Ferziger, J., *Phys. Fluids* **13**, 1281 (2001).