Anomalous Dissipation and Spontaneous Stochasticity in Surface Quasi-Geostrophic Flow

N. Valade[a], S. Thalabard[b], J. Bec[a,b]

Surface Quasi Geostrophic equations (SQG) describe the two-dimensional active transport of a temperature field in a strongly stratified and rotating environment. Besides its relevance to geophysics, SQG bears formal resemblance with various flows of interest for turbulence studies, from passive scalar and Burgers to incompressible fluids in two and three dimensions.

This analogy is here substantiated by considering the turbulent SQG regime emerging from deterministic and smooth initial data prescribed by the superposition of a few Fourier modes. While still unsettled in the inviscid case, the initial value problem is known to be mathematically well-posed when regularized by a small viscosity. In practice, numerics[2] reveal that in the presence of viscosity, a turbulent regime appears in finite time, which features three of the distinctive *anomalies* usually observed in three-dimensional developed turbulence : (i) dissipative anomaly, (ii) intermittency, and (iii) super-diffusive separation of fluid particles, both backward and forward in time. These three anomalies point towards three spontaneously broken symmetries in the vanishing viscosity limit: time reversal, scale invariance and uniqueness of the Lagrangian flow, a fascinating phenomenon that Krzysztof Gawedzki[1] dubbed *spontaneous stochasticity*.

Combing insights from both the direct numerical simulations and simplified models, we argue that spontaneous stochasticity and irreversibility are the two sides of a single coin in SQG[2]. Our numerics, though, reveal that the deterministic SQG setting only features a *tempered* version of spontaneous stochasticity, characterized in particular by non-universal statistics.

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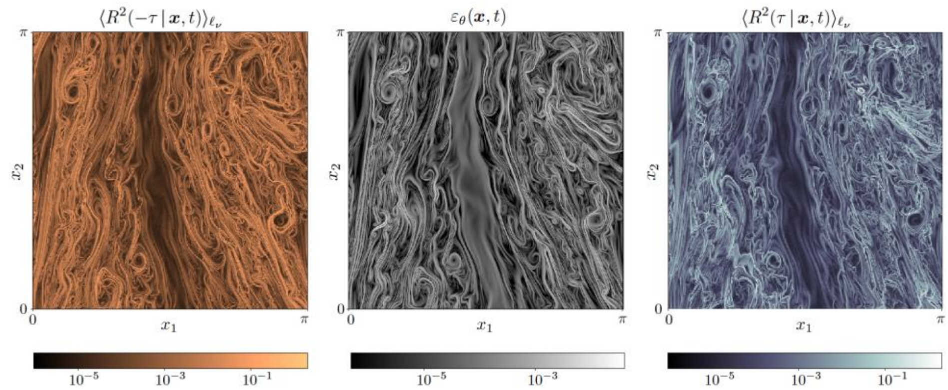
[a] Calisto, INRIA Sophia Antipolis, Valbonne, France
[b] Inphyni , Université Côte d'Azur, Nice, France
[1] Gawedzki, Intermittency in Turbulent Flows., Cambridge (2001)
[2] Valade et al., *Annales Henri Poincaré*, 1-23 (2023).



Left/right: Backward/forward mean-squared separation of tracers as a function of $\boldsymbol{x}$.
Centre: Scalar temperature dissipation fields.