Instability triggering by acoustic waves in a turbulent compressible shear layer

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Our modern understanding of acoustic noise emission is closely linked to the triggering of acoustic instabilities. We therefore analyze the acoustic processes in a turbulent shear layer with hyperbolic tangent main flow profile[1] and link acoustic instabilities to the reflection and transmission of acoustic waves, i.e. to the receptivity of the shear layer. The analysis reveals a close link between the so-called critical layer, where the phase velocity of an acoustic wave and the main flow velocity coincide, and the most unstable acoustic mode. Our analysis is based on an exact solution we found for the Pridmore-Brown equation[2], resulting from the system of linearized compressible Euler equations. Due to the inviscid consideration of the acoustic waves, the problem describing Pridmore-Brown equation comes with a number of singularities, which lead to to the exact solution in terms of the general Heun function[3] for arbitrary Mach numbers. By means of this solution, we formulate a temporal eigenvalue problem, whose solution describes the growth of acoustic waves in the turbulent main flow. An exemplarly set of eigenvalues can be seen in Fig. 1. Further, we investigate an acoustic wave hitting the shear layer and answer the question of reflection and transmission of this wave by means of the general solution. Linking both results, we show that the critical layer as the key property of the flow is decisive for both the energy exchange between an incident acoustic wave and the shear layer and for the generation of unstable modes. We therefore state that the absorption of certain acoustic waves leads to acoustic instability triggering and thus noise generation. In this regard, we give a criterion for an incident acoustic wave being partially absorbed and triggering an associated acoustic instability.

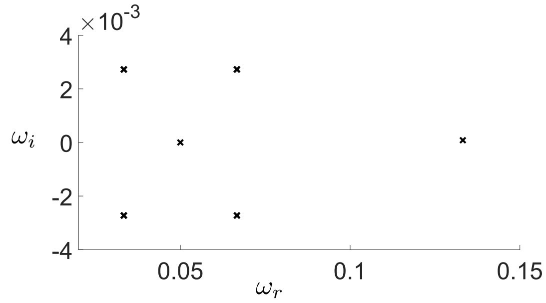

Figure 1: Pair of dominant fast and slow unstable acoustic eigenvalues for Mach number $M = 3$ and wavenumber $\alpha = 0.1$.

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[1] Sharan et al., *J. Fluid Mech.* **877**, 35 (2019).
[2] Pridmore-Brown, *J. Fluid Mech.* **4**, 393 (1958).
[3] Ronveaux, *Heun's Differential Equations*, Oxford (1995).