Influence of wall impedance and temperature on the stability of turbulent high Mach number boundary layers

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Unstable acoustic modes in high Mach number boundary layer flows contribute to the laminar-turbulent transition and play a major role in noise emission of a developed turbulent flow. We therefore investigate the influence of acoustic wall linings and wall cooling or heating on the acoustic modes of high-velocity turbulent boundary layers, modeled by the exponential similarity solution as turbulent main flow profile. We calculate the full spectrum of eigenmodes under variation of the wall temperature T_w and the acoustic wall impedance Z, which describes the acoustic effect of wall linings by coupling the pressure fluctuation $\hat{p}$ and the wall-normal velocity fluctuation $\hat{v}$ at the wall in the frequency domain through[1] $\hat{p}_w(\omega) = Z(\omega)\,\hat{v}_w(\omega)$. Our stability analysis is based on the compressible Rayleigh equation[2] resulting from the linearized Euler equations, to which we derive an exact solution given in terms of the general Heun function. This analytic solution together with the impedance boundary condition at the wall and the far-field condition provides us an algebraic equation, which allows the calculation of all eigenvalues depending on the parameters T_w and Z. Further, the exact solution yields an explicit analytic description of the eigenfunctions.

Our analyses reveal a strong influence of wall impedance on the stability behavior of the acoustic modes and thus on turbulent noise generation. The change of the temporal growth rate of the most unstable second-mode disturbance under variation of wall impedance and wall-to-free stream temperature ratio T_w/T_∞ is shown in Fig. 1. Regarding the wall temperature impact, our results based on the analytical solution are consistent with numerical literature results, stating that wall cooling stabilizes the first mode but destabilizes higher modes[3]. Since the algebraic eigenvalue equation can be easily extended to any frequency-dependent impedance model $Z(\omega)$, we also show the damping effect of different impedance models on unstable acoustic modes. The investigations thus contribute to the suppression of turbulent boundary layer noise.

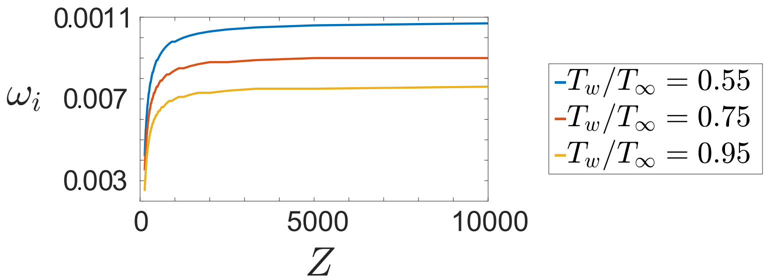

Figure 1: Second-mode's growth rate ω_i for different wall impedances and temperatures.

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[1]Rienstra and Hirschberg, *An Introduction to Acoustics*, Eindhoven (2020).
[2]Criminale et al., *Theory and Computation in Hydrodynamic Stability*, Cambridge (2018).
[3]Mack, *Boundary layer stability theory*, Pasadena (1984).