Integral identities for the study of the skin-friction coefficient of incompressible and compressible boundary layers

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We show that the integral relation discovered by Fukagata et al. (2002)[1] for the skin-friction coefficient of free-stream incompressible boundary layers simplifies to the von Kármán momentum integral equation when the upper integration bound along the wall-normal direction is taken asymptotically large (Ricco & Skote (2022)[2]). If the upper bound is finite, the weighted contributions of the terms of the streamwise momentum equation depend spuriously on the bound itself. Analogously, the compressible versions of the Fukagata et al. (2002) identity reduce to the compressible von Kármán momentum integral equation. We discuss how the upper integration bound influences the physical interpretation of the terms in these compressible identities. We also prove that the family of infinite identities obtained by successive integrations also reduces to the von Kármán equation and it degenerates to the definition of skin-friction coefficient as the number of integrations grows asymptotically. The dependence on the number of repeated integrations is therefore also non-physical. We close our discussion by presenting the extension of the integral identity found by Elnahhas & Johnson (2022)[3] for incompressible boundary layers to the compressible regime, showing how the contributions of the laminar skin-friction coefficient and the Favre-Reynolds stresses can be isolated (Xu et al., 2023[4]).

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