Turbulent Rankine-Hugoniot Relations

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Recent shock simulations in the hydrodynamics code, xRage, coupled to a RANS model, BHR [1], showed a peculiar result. They demonstrated that the initialization of a shock with the classical Rankine-Hugoniot relations [2], introduced an initial turbulent kinetic energy field, k_0, dependence on the shock speed. Figure (1) shows velocities as a function of time for a fixed point in space as a shock translated past it.

In these simulations, the gas was Argon with a Mach 3.338 shock in the absence of turbulence. However k_0 initialized as homogeneous, isotropic turbulence, changed the shock speed. The colored curves represent different initial conditions for turbulent kinetic energy, with green, $k_0 = 10^8$ (cm^2 s^{-2}); red, $k_0 = 10^7$ (cm^2 s^{-2}); and blue, $k_0 = 10^0$ (cm^2 s^{-2}), a negligible k_0. The analytical solution, black, is without turbulent motions. Arrival of the shock is indicated by the initial rise of each curve.

The naive application of the classical Rankine-Hugoniot relations result in shock speeds that increase with k_0. In this study we re-derive the Rankine-Hugoniot shock relations to include the presence of turbulence in the background flow field such that the desired shock speed is attained and it does not depend on k_0,

$$\overline{\rho}_1 \tilde{u}_1 = \overline{\rho}_2 \tilde{u}_2, \tag{1}$$

$$\overline{\rho}_1 \tilde{u}_1^2 + \overline{p}_1 + \frac{2}{3}\overline{\rho}_1 \tilde{k}_1 = \overline{\rho}_2 \tilde{u}_2^2 + \overline{p}_2 + \frac{2}{3}\overline{\rho}_2 \tilde{k}_2, \tag{2}$$

$$\overline{\rho}_1 \tilde{E}_1 \tilde{u}_1 + \overline{p}_1 \tilde{u}_1 + \frac{2}{3}\overline{\rho}_1 \tilde{k}_1 \tilde{u}_1 = \overline{\rho}_2 \tilde{E}_2 \tilde{u}_2 + \overline{p}_2 \tilde{u}_2 + \frac{2}{3}\overline{\rho}_2 \tilde{k}_2 \tilde{u}_2. \tag{3}$$

In equations (1), (2) and (3), ρ is the density, u is the velocity normal to the shock, p is the pressure and $E = e + \frac{1}{2}u^2$ is the specific total energy, where e is specific internal energy. The state upstream of the shock is indicated by the subscript 2 and the downstream state are the variables with subscript 1. The turbulent Rankine-Hugoniot relations were derived from the compressible Euler's equations with a Reynolds stress.

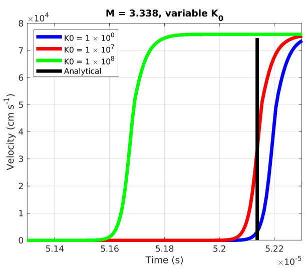

Figure 1: Velocity behind the shock, modeled and analytical solution.

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[1] Schwarzkopf et al., *Flow Turbul. Combust.* **96**, 1 (2016).

[2] Anderson, *Modern Compressible Flow: with Historical Perspective* (1990).