Helical Turbulence - the Transition between 2D and 3D Turbulence

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Our theoretical understanding of the differences between 2D and 3D turbulence, based on the concept of vortex stretching, shows fundamental differences. The energy transfer in 3D turbulence occurs from large to small scales, whereas in 2D turbulence it is the opposite and occurs from small to large scales. To better grasp the differences and transitions between 2D and 3D turbulence, helical-symmetric flows are investigated, which are common in technical equipment such as wind turbines, combustion chambers, and marine propellers. The helical coordinate system $(r,\, \zeta,\, \eta)$ is given by r, $\zeta = az + b\phi$ and $\eta = -bz + ar^2\phi$, where a, $b = \text{const}$, $a^2 + b^2 > 0$ and (r, ϕ, z) are the common cylindrical coordinates.

Helical symmetry implies, all dependent variables are independent of η, i.e. helically reduced Navier-Stokes equations emerge with $\frac{\partial}{\partial \eta} \equiv 0$. It should be pointed out that all three velocity components $(u^r,\, u^\zeta,\, u^\eta)$ are non-zero and develope on a 2D (r,ζ)-manifold. Helical flows differ whether they have a velocity u^η along the helix or not, where a non-zero velocity u^η induces vortex stretching[1]. Furthermore, in both cases, helical flows admit infinite classes of conservation laws and thus integral invariants exist. The central new invariants for helical turbulence are generalized helicity, when vortex stretching is present, and generalized enstrophy, without vortex stretching. The findings from 2D and 3D turbulence show that global invariants play a central role in turbulence and this is also expected for helical turbulence. Appropriate large-scale simulations are conducted for this purpose.

The helically reduced Navier-Stokes equations are discretized using high-order discontinous Galerkin (DG) scheme[2]. Following the ideas of Khorrami et al. (1989)[3], uniqueness conditions at the centerline axis for $r = 0$ are implemented with a change of the polynomial basis. A third-order semi-explicit method is used for the temporal integration[4], separating the spatial operator into the Stokes operator, which is discretized implicitly, and the operator for nonlinear terms, which is treated explicitly.

This numerical framework is utilized to study energy transport in helically symmetric flows for high Reynolds numbers. Additionally, the transition from 2D to 3D turbulence is investigated by analyzing asymptotically small velocities along the helix and the resulting asymptotically small vortex stretching. Results and comparison of invariant theory and high-fidelity simulation of helical turbulence will presented at the meeting.

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