Reynolds number induced growth of large-scale rolls in plane Couette flow and invariant scaling laws for added wall-transpiration using resolvent analysis

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In DNS of turbulent Couette flow the observation has been made, that the long streamwise rolls increase in length with the Reynolds number [1]. For its understanding we employ the resolvent analysis emphasising the high Reynolds number ($Re \to \infty$) and small streamwise wavenumbers ($\alpha \to 0$) limit imposing the distinguished limit $Re_\alpha = Re \cdot \alpha = O(1)$ as done for the linear stability analysis of the asymptotic suction boundary layer flow [2], since structures are only significantly amplified for high Reynolds number plane Couette flow, which admit very weak streamwise variation. We find that Re_α acts as a local invariant in the behaviour of the energy of the system characterised through the first singular value σ_1 of the resolvent operator within the investigated asymptotic limit, where the behaviour of σ_1 over both Re and α respectively stays invariant within a certain Re_α-range. In order to obtain constant streamwise structures for increasing Reynolds numbers, the respective streamwise wavenumber has to decrease, which verifies the observations from DNS studies of an increasing length of the streamwise structures with the Reynolds number.
The conducted analysis is expanded on the plane Couette flow with constant wall-transpiration velocity V_0 and wall velocity U_w with emphasis on the effect of V_0 on the coherent structures. For this flow setting, the first singular value σ_1 for a constant ratio of the wall-transpiration and streamwise Reynolds numbers $\gamma = \frac{Re_{V_0}}{Re} = \frac{V_0}{U_w}$ is the largest for an invariant relationship $Re \cdot \gamma^a = C$. The first singular value σ_1 reaches a peak value for a specific Re for each γ characterised through the aforementioned invariant relationship. The influence of the wall-transpiration Reynolds number Re_{V0} on the streamwise structures is analysed, where it is shown that the streamwise structures not only move closer to the upper wall, but also become more confined in both the wall-normal and spanwise direction for an increasing Re_{V0} while keeping the streamwise Reynolds number Re constant.

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