Logarithmic lattice models for flows with boundaries

Ciro S. Campolina* and Alexei Mailybaev†

Important problems for the theory of turbulence are related to the spontaneous development of localized structures in the vicinity of boundaries. This is the case of finite time singularities at walls in ideal flow, the boundary layer detachment, and the convergence of the Navier-Stokes solutions to Euler's. While those mathematical questions remain open, numerical simulations play an important role in their investigation. Despite the great advances in recent years, the limited resolution of current computational techniques appears to be insufficient for the accurate investigation of the finest structures of the flow.

As an alternative way, it has been proposed to model extremely fine scales by considering Fourier wave vectors in logarithmic lattices[1]. In this framework, the resulting models are structurally the same as the original ones, they preserve the group of symmetries and invariants, but can be simulated over impressively large spatial ranges. The technique has been successfully applied to fluids filling the whole Euclidean space[2]. However, since only Fourier variables are available, it is not straightforward how to consider boundaries within the flow.

In this talk, we show how to model flows with boundaries in the logarithmic lattice formalism. The strategy is to model the wall as a discontinuity surface immersed in the flow, which is extended to the whole euclidean space by a mirror symmetry. In this way, we can work with Fourier variables, and so with logarithmic lattices. The discontinuities in the field variables give rise to localized shear forces in the governing equations, which can be exactly solved from the system. We show how this approach reproduces some properties of classical shear flows. When applied to the 2D Navier-Stokes equations, we reach incredibly large Reynolds numbers and have access to extremely fine scales of the flow. We verify a transition to turbulence and the appearance of sharp bumps in energy dissipation. Despite the incredibly high Reynolds numbers from our simulations, we argue that is still difficult to give a proper answer about the possible convergence to the Euler flow.

*Université Côte d'Azur, Inria, CNRS, Sophia-Antipolis, France
†Instituto Nacional de Matemática Pura e Aplicada, IMPA, Rio de Janeiro, Brazil

[1] Campolina and Mailybaev, *Nonlinearity* **34**, 4684 (2021).
[2] Campolina and Mailybaev, *Phys. Rev. Lett.* **121**, 064501 (2018).