Mutual friction coupling of Gross-Pitaevskii and Navier-Stokes equations

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Mutual friction produced by the interaction of the normal fluid component with the superfluid vortices is a phenomenon of paramount importance in the field of finite-temperature superfluid turbulence. It is described in the HVBK model through a mutual friction interaction between the normal Navier-Stokes (NS) fluid and the superfluid modelled by a coarse-grained and rotational NS superfluid velocity.[1] Mutual friction can also be described, in a more mesoscopic way, in the vortex filament model through an interaction between the normal NS fluid and the vortex line dynamics described by Biot-Savart interactions.[2,3]

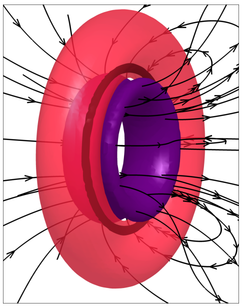

Figure 1: Triple-vortex flow generated by a super-fluid vortex ring (black) moving in a normal (viscous) fluid.

We present in this contribution a new model to couple a Navier-Stokes incompressible fluid with a Gross-Pitaevskii superfluid. Coupling terms in the global system of equations involve new definitions of the following concepts: the regularized superfluid vorticity and velocity fields, the friction force exerted by quantized vortices to the normal fluid, the covariant gradient operator in the Gross-Pitaevskii based on a slip velocity respecting the dynamics of vortex lines in the normal fluid.

A numerical algorithm based on pseudo-spectral Fourier methods was developed to solve the coupled system of equations. The new code was tested against well-known benchmarks for the dynamics of different types or arrangements of quantized vortices (vortex crystal, vortex dipole and vortex rings) evolving in a normal fluid. The evolution of superfluid vortex rings in a normal fluid (see figure) and the head-on collision of vortex rings will be presented in detail.

The new coupling model[4] has the advantage to keep the full Gross-Pitaevskii dynamics for the superfluid, and thus avoiding any phenomenological models for vortex nucleation or reconnection. This opens new possibilities to revisit and enrich existing numerical results for complex quantum fluids. We also present recent results obtained using this model to simulate counter-flow or co-flow quantum turbulence initiated by Taylor-Green vortices.[5]

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