Extending the Gibbon-Fokas-Doering stagnation-point-type ansatz to finite-energy initial conditions: A solution to the Navier-Stokes Millennium Prize Problem?

M. D. Bustamante*

The stagnation-point-type solution to the 3D incompressible Navier-Stokes equations found in[1] produced an infinite family of solutions to the 3D incompressible Euler equations that blow up in a finite time. There is an exact formula for the singularity time as a functional of the initial conditions[2,3,4], and the solutions to this and related models are best understood in terms of infinitesimal Lie symmetries[5]. The main drawback of these solutions, *from the viewpoint of the Clay Millennium Prize*, is that the velocity field depends linearly on the out-of-plane spatial coordinate, and thus the initial condition has infinite energy. In this talk, I will present a way to extend these solutions in order to have an arbitrary dependence on the out-of-plane coordinate, allowing in principle for finite-energy solutions. This extension seems to break the infinitesimal Lie symmetry structure inherent to the previous infinite-energy solutions, so a statement regarding finite-time blowup is not yet available analytically in the finite-energy case. However, the extension allows for a novel numerical attempt at the finite-energy solution, via a hierarchy of systems of coupled 2D partial differential equations, which are much easier to handle than a full 3D problem. I will present results and prospects, and discuss potential applications to real-life experiments.

To illustrate the challenges faced, figure 1 shows snapshots of a well-resolved simulation corresponding to a scenario of helical vortices with theoretically no finite-time blowup, but whose small scales become unresolved eventually at any given resolution.

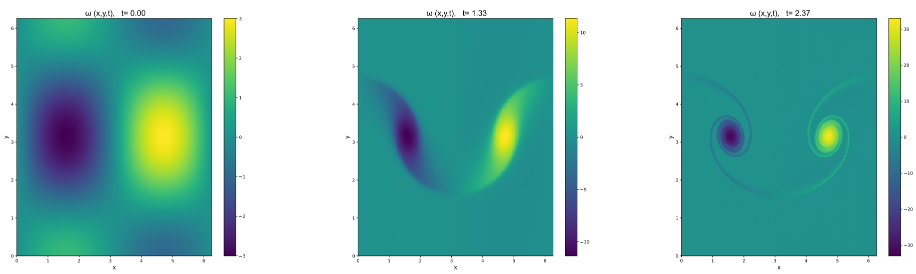

Figure 1: Snapshots, at different times, of the symmetry-plane vorticity field of a pseudospectral simulation ($N = 256^2$) of the 3D Euler fluid equations (Gibbon-Fokas-Doering ansatz), for the case of initial conditions $\gamma_0 = \sin(y - \pi/2)$ (vorticity stretching rate) and $\omega_0 = [1 + 2\sin(y - \pi/2)]\cos(x + \pi/2)$ (vorticity).

*School of Mathematics and Statistics, University College Dublin, Belfield, Dublin 4, Ireland

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