Comparison of phase-field method formulations for simulations of bubbly flows

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Multiphase flows are ubiquitous in our daily life, from wastewater treatment and pharmaceutical industries to oil and nuclear industries. Augmenting the efficiency of these industrial processes is paramount for reducing pollutant emissions and hindering climate change. Numerical simulations of multiphase flows are non-trivial since turbulence and non-linear interfacial phenomena occur at the same time on a wide range of scales (from the molecular up to the inertial scale). In this context, the phase-field methods¹ represent a powerful tool capable of approximating the interface behaviour and naturally capturing coalescence and breakage phenomena. In addition, single-phase solvers can be easily adapted to solve multiphase problems with great scalability properties. The classic phase-field method based on the conservative Cahn-Hilliard (CH) equation underlies a strong thermodynamic basis. Besides these positive aspects, the numerical discretisation of the fourth-order derivative involved in the CH equation is challenging. An alternative approach is represented by the use of the conservative Allen-Cahn (AC) formulation², which only employs derivatives of the second order and hence reduces the numerical complexity. In this work, we compare the two formulations (CH and AC) in two standard benchmarks. The first is a bubble in a shear flow from which we can address the ability of the models to reproduce the analytical solution. The second is a rising bubble in a quiescent fluid where breakage due to shear stresses is allowed. This last test amplifies the mass leakage phenomena, hence it is optimal for comparing the performance of the two approaches. Addressing the capability of the new AC method could allow further investigations of the effects of a bubbles swarm in wall-bounded turbulence compared to the current implementation³. In other words, numerical simulations with more realistic density and viscosity ratios.

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¹Soligo et al., J. Fluids Eng. **143**, 080801 (2021) ²Mirjalili et.al, J. Comput. Phys. **401**, 109006 (2020)

³Mangani et al., *Phys. Rev. Fluids* **7**, 053601 (2022)