Balanced non-equilibrium turbulence

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A landmark of turbulence research remains Kolmogorov's K41 theory, regulating many quantities of engineering interest. K41 is based on the assumption of cascade equilibrium, which, in unsteady turbulence, is expected to be fulfilled as Reynolds numbers and eddy wavenubers both tend to infinity - it is therefore valid only in an asymptotic sense [1]. Here, I present a generalization of the K41 for the finite wavenumbers of homogenous decaying turbulence far from initial conditions, where weak non-equilibrium effects act. The predictions are validated using direct numerical simulations (DNS) and hot wire anemometry (HWA) measurements.

The novel theory is based on two arguments. *First*, that the interscale energy flux evolves in a self preserving manner, i.e. $\Pi(k,t) = \epsilon(t)g(kL)$, where Π is the interscale energy flux, ϵ the total dissipation, k the wavenumber and L the integral length scale. *Second*, that there exists a range of scales in the cascade where energy is simply transported at higher wavenumbers. Mathematically this is expressed as $D\Pi/Dt = 0$ where D/Dt denotes the substantial derivative in the normalized wavenumber-time coordinate system. Both of the above assumptions are well-supported from the DNS [2]. On the basis of the above two assumptions, the following finite-wavenumber generalization of the -5/3 law can be derived

$$E(k,t)/(\epsilon(t)^{2/3}L(t)^{5/3}) = C_k[1 - c\kappa^{-2/3}]^2\kappa^{-5/3}\,, \qquad (1)$$

where $\kappa = kL$, E is the three dimensional energy spectrum, C_k the Kolmogorov constant, and c a constant depending on the dissipation constant and the chosen turbulence invariant. The above scaling is well supported by the compensated spectra of both DNS and HWA presented in figure 1.

(a) (b)

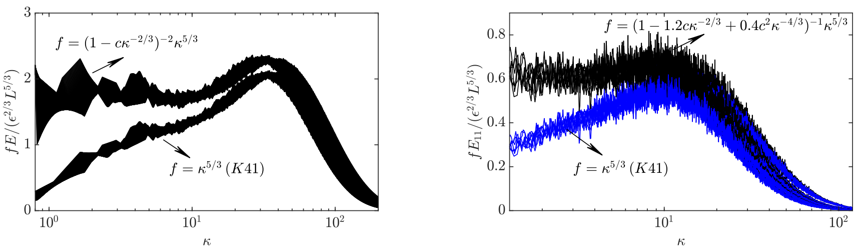

Figure 1: Validation of equation 1 using (a) DNS and (b) HWA of homogenous decaying turbulence. The novel scaling produces flatter compensated spectra at lower wavenumbers, compared to the standard -5/3 law. At (b) a transformation of equation 1 for one-dimensional spectra is used.

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[1] Pope SB, Turbulent flows. Cambridge University Press (2000).

[2] Steiros K, Phys. Rev. E **105**:3, 035109 (2022).