Perron–Frobenius analysis of bursting in wall turbulence

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A well-known model for the intermittent bursting of wall-bounded turbulence is the transient amplification of the structures of the wall-normal velocity as they are tilted by the shear[1]. This can be represented as a clockwise path along the top of a two-dimensional, roughly triangular, pdf of flow states in an inclination-amplitude parameter space (Fig. 1a). However, this model does not include a mechanism for burst regeneration, which would close the path in state space.

The Perron-Frobenius operator describes probability transfer among different times[2]. It allows the identification of likely precursors and effects of particular states within the pdf, and leads to entropy-based measures of coherence that assist in the choice of variables for reduced system representations. When applied to an $Re_\tau \approx 10^3$ small-box channel, it can be used to show that the precursors for the re-initiation of the burst from the lower left-hand corner of the triangular pdf lie along the bottom of the triangle (Fig. 1b), and to provide an estimate of the time required by this process. It also identifies trajectories in state space that lead to the burst re-initiation. Conditional averaging along them suggests that regeneration is part of a mutual interaction between bursting and the mean velocity profile. The most likely regenerative state contains a low-shear region near the wall and a high-shear layer above it.The new bursts are generated from the latter[3] (Fig. 1c). The direction of causality can be confirmed by numerical experiments in which the effect of the bursts on the mean profile is modified, with little effect on the burst dynamics.

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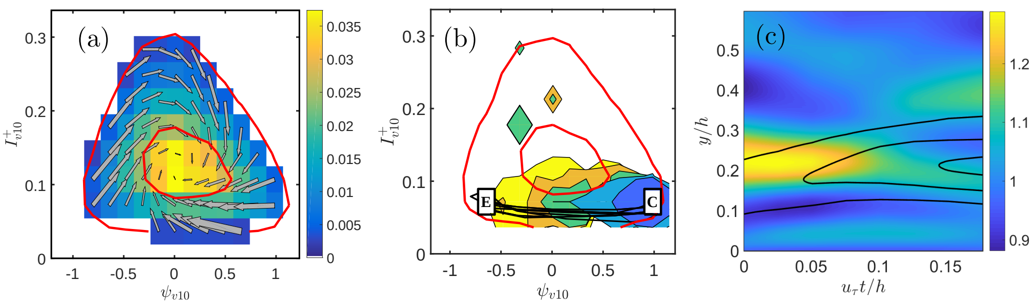

Figure 1: (a) Pdf of inclination and intensity of the wall-normal velocity in a small channel. Arrows are the average flow of the state variables. (b) As in (a). Filled contours are probability densities at $u_\tau t/h = 0.025(0.025)0.125$ of states initially in cell "C" at $t = 0$. The black lines are state trajectories connecting "C" to "E". (c) Average temporal evolution of the flow profiles along the trajectories in (b). Shading is (x, z)-integrated shear, normalised with its long-time average. Line contours are perturbation kinetic energy.

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[1] Jiménez, *Phys. Fluids* **27**, 065102 (2015).

[2] Beck and Schlögl, *Thermodynamics of chaotic systems*, Cambridge (1993).

[3] Jiménez, `https://arxiv.org/abs/2301.08948` (2023).