Search for unstable relative periodic orbits in plane Poiseuille flow using symmetry-reduced dynamic mode decomposition

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Within the dynamical systems approach to turbulence, state-space structures such as unstable periodic orbits (UPOs) embedded in the chaotic attractor are known to carry information about the dynamics of the system and therefore provide a tool for its description. UPOs are usually found by Newton searches, and constructing good initial data is challenging. A commonly used technique to generate such initial data involves detecting recurrence events by comparing future with past flow states using their L_2-distance[1]. A drawback of this method is the need for the trajectory to shadow the UPO for at least one of its cycles, which becomes less likely for higher Reynolds numbers. Furthermore, one only obtains local-in-time information. A method that bypasses both issues is based on dynamic mode decomposition (DMD), where initial guesses are constructed using a few dynamic modes corresponding to dominant frequencies identified from the data[2]. However, in the presence of continuous symmetries, DMD fails to provide accurate approximations of the dynamics. To address this, we combine symmetry-reduced DMD (SRDMD), an approach recently developed by[3] with sparsity promotion[4]. In doing so, we construct optimal low-dimensional representations of the data that are global objects, and each instant in time can be chosen as an initial guess. We apply the method to data obtained by direct numerical simulation of 3d plane Poiseuille flow at friction Reynolds number $\mathrm{Re}_\tau = 50$, taking a shift symmetry in streamwise direction explicitly into account. We demonstrate, that the obtained unstable relative periodic orbits (URPOs) cover relevant regions of the systems state space, suggesting their importance for a possible description of the flow.

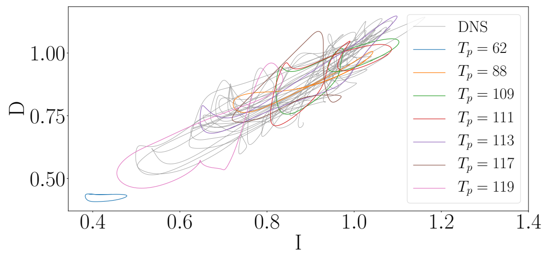

Figure 1: State space in a two-dimensional projection of energy input versus dissipation, normalised by their laminar flow values. The DNS time series is shown in grey. The coloured lines show the first cycle of each URPO.

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