# The role of large surface roughness in transition of three–dimensional boundary layers

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Surface roughness can influence significantly transition of three–dimensional boundary layers through different mechanisms including receptivity[1] and alteration of the base flow whereby modifying crossflow stability[2] or even inducing new instabilities.[3] We focus on the second mechanism, studying the impact of surface roughness on transition of three–dimensional boundary layers, represented by the Falkner–Skan–Cooke flow. Surface roughness is in the form of a wavy wall and can have a height of $\mathcal{O}\left(R^{-1/3}\delta\right)$, where the Reynolds number $R$ based on the local boundary layer thickness $\delta$. Despite being much smaller than $\delta$, the height of the roughness is large enough to induce fundamentally nonlinear response.

The flow structure can be characterized by a wall layer, the main layer, and a critical layer. First, the immediate response to the roughness is in a viscous wall layer with an $\mathcal{O}\left(R^{-1/3}\right)$ width, introduced to satisfy the no–slip condition on the surface. The wall layer is also nonlinear because the surface roughness height is comparable with its width. The wall layer converts the surface displacement into a 'blowing velocity' of $\mathcal{O}\left(R^{-2/3}\right)$ into the main layer. The nonlinear streaming effect in the wall layer leads to a mean flow distortion of $\mathcal{O}\left(R^{-1/3}\right)$.

The main layer occupies the bulk of the boundary layer, where the flow field can be decomposed into the base flow, the steady streaming of $\mathcal{O}\left(R^{-1/3}\right)$ and the forced perturbations of $\mathcal{O}\left(R^{-2/3}\right)$. The former is described by an initial–value problem, while the latter is governed by the Rayleigh equation with the 'blowing velocity' induced by the wall layer acting as the lower boundary condition. The Rayleigh equation becomes singular at a position denoted as the critical level.

The singularity is resolved by introducing viscous effects in the critical layer with an $\mathcal{O}\left(R^{-1/3}\right)$ width, and the regularised disturbance velocity acquires a larger amplitude of $\mathcal{O}\left(R^{-1/3}\right)$, while the vorticity becomes $\mathcal{O}\left(1\right)$. The critical layer is strongly nonlinear so that all harmonics are generated at the same order. There are velocity jumps across the critical layer, which couple the dynamics in the critical layer and the main layer. The streaming effects in the critical layer and the wall layer act as the main forcing for the initial–value problem governing the mean–flow distortion in the main layer. Numerical solutions show that nonlinearity enhances the response. A new instability may arise as a result.

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