Characterising spontaneous stochasticity in turbulence with path entropy

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Chaotic systems are characterised by exponential separation between close-by trajectories, which in particular leads to deterministic unpredictability over an infinite time-window. It is now believed, that such butterfly effect is not fully relevant to account for the type of randomness observed in turbulence. For example, tracers in homogeneous isotropic flows are observed to separate algebraically, following a universal cubic growth, independent from the initial separation. This regime, known as Richardon's regime, suggests that at the level of trajectories, and unlike in chaos theory, randomness may in fact emerge in finite-time. This phenomenon called "spontaneous stochasticity" originates from the singular nature of the underlying dynamics, and provides a candidate framework for turbulent randomness and transport.

While spontaneous stochasticity has been mathematically formalised in simplified turbulence models[1 2], a precise and systematic tool for quantifying the various facets of this phenomenon is to this day missing.

We here characterise the emergence of spontaneous stochasticity through the use of an entropy defined at the levels of lagrangian trajectories. This path entropy is a finite-time counterpart to the standard Kolmogorov-Sinai entropy used in chaos theory, which there connects to fractal dimensions of strange attractors[3]. In a finite-time setting, we argue that this tool is relevant to capture spontaneously stochastic scenarios. We show indeed, through a range of models with increasing complexity, that it bears information of the underlying randomness. This leads us to distinguish between various levels of spontaneous stochasticity.

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[1]Mailybaev, *Multiscale Modeling and Simulation (SIAM)* **14**, 96-112 (2016)
[2]Mailybaev and Raibekas, *Arnold Mathematical Journal* (2022)
[3]Mañé, *Springer-Verlag*, (1987)