Enstrophy flux and physical-space locality in three-dimensional turbulence

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In three-dimensional turbulence, the governing equation for the enstrophy spectrum $E_\omega(k) \equiv \int \frac{1}{2}\hat{\omega}_i\hat{\omega}_i^* \, \mathrm{d}\Omega_k$ reads

$$\frac{\partial E_\omega}{\partial t}(k) = \underbrace{P_\omega(k)}_{\text{Injection}} + \underbrace{\int \widehat{\omega_j \partial_j u_i}\hat{\omega}_i^* \, \mathrm{d}\Omega_k}_{\text{Production}} - \underbrace{\int \widehat{u_j \partial_j \omega_i}\hat{\omega}_i^* \, \mathrm{d}\Omega_k}_{\text{Transfer}} - \underbrace{2\nu k^2 E_\omega(k)}_{\text{Dissipation}}, \tag{1}$$

where $\hat{(\cdot)}$ and $(\cdot)^*$ denote the Fourier transform and the complex conjugate, respectively, along with the spherical integral $\mathrm{d}\Omega_k$ and the kinematic viscosity ν. Due to the enstrophy production term associated with the vortex stretching, enstrophy is not an inviscid invariant. Figure 1 (a) shows the enstrophy budget with the integral $\int_0^k \mathrm{d}k'$ of the last three terms on the RHS of Eq. 1. In the inertial range, enstrophy is amplified by the production term and transported to smaller scales by the transfer term associated with vortex advection[1]. The dissipation term drains the enstrophy in the higher wavenumbers. For Navier–Stokes turbulence without spatially nonlocal nonlinear interactions[2], we find in Fig. 1 (b) that the enstrophy production is suppressed for scales larger than R, resulting in a close-to-constant profile. In the presentation, we will discuss the nature of the somewhat overlooked enstrophy cascade and its physical-space locality in three-dimensional turbulence.

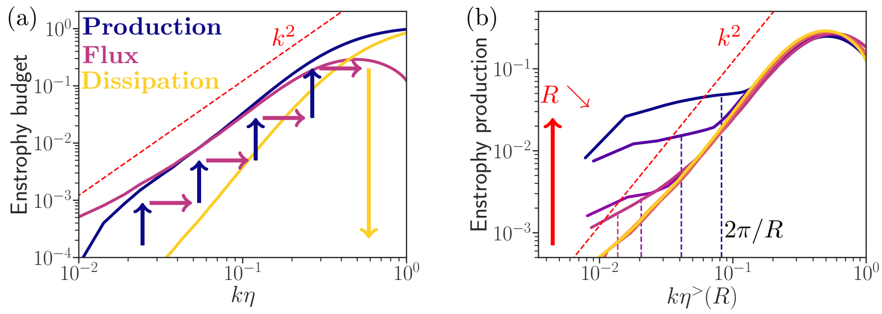

Figure 1: (a) Scale-by-scale enstrophy budget. (b) The enstrophy flux spectrum in space-local Navier–Stokes turbulence. Darker (lighter) colour denotes the smaller (larger) value of R: the radius of the space-local filter. The vertical dashed lines denote the corresponding length scale $2\pi/R$. For both panels, wavenumber k is normalised by the Kolmogorov length scale η.

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[1] Davidson, Morishita and Kaneda, *J. Turbul.* **9**, N42 (2008).
[2] Araki, Bos and Goto, "Space-local Navier–Stokes turbulence" (in preparation).