Exact invariant solutions and their bifurcations in transitional vertical natural convection

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Thermal convection in a three-dimensional differentially heated vertical channel is numerically investigated in the transitional regime where the flow is known to exhibit spatio-temporally rich dynamics and self-organized coherent patterns. Previous numerical simulations[1,2] reveal a variety of behaviors – steady, time-periodic and chaotic. We extend this work by constructing exact invariant solutions such as equilibria and periodic orbits of the underlying Oberbeck–Boussinesq equations, and compute their bifurcating stable/unstable branches by parametric continuations.

A sequence of local bifurcations (including pitch-fork, saddle-node, Hopf and period-doubling) take place from the unstable featureless base state to various modulated convection-roll-dominated turbulent states. We identify two global bifurcations, first, a homoclinic cycle from modulated transverse rolls (FP1, Figure 1(a)) and second, a robust heteroclinic cycle linking two symmetry-related fixed points capturing a wavy-roll pattern (FP2 & $\tau(0.25, 0)$FP2, Figure 1(b-c)). These global bifurcations are confirmed by phase portraits (Figure 1(c)) as well as the logarithmic divergence of the period close to the bifurcation point.

Our work highlights the interest of fully non-linear invariant solutions that can explain the origin of the flow patterns far above the threshold, where standard linear stability analysis and weakly non-linear methods are not applicable.

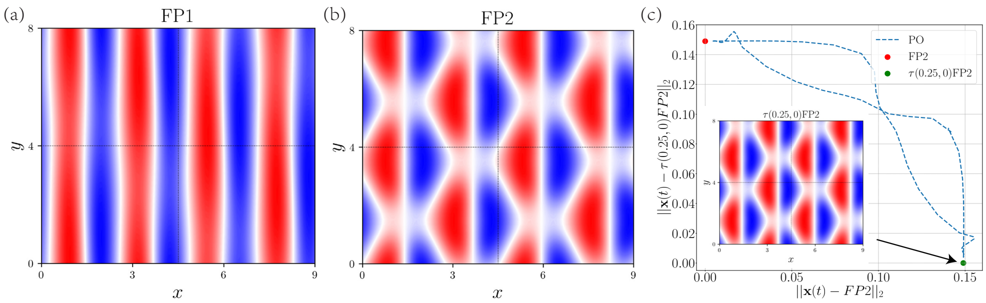

Figure 1: (a-b) Snapshots of modulated transverse rolls (a, FP1) and wavy rolls (b, FP2), from which homoclinic and hetero-clinic cycles bifurcate. (c) State space projection of the heteroclinic cycle close to the global bifurcation point, with the spatially shifted FP2 ($\tau(0.25, 0)$FP2) in the inset.

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[1] Gao et al., *Phys. Rev. E* **88**, 023010 (2013).

[2] Gao et al., *Phys. Rev. E* **97**, 053107 (2018).