Capturing the edge of chaos as a spectral submanifold in pipe flows

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Unstable exact coherent states (ECSs) are thought to form the core of turbulence, as has been demonstrated in canonical shear flow configurations. From a dynamical system point of view, the unstable and stable manifolds of these distinguished solutions organize the geometry of the phase space. An extended turbulent state can coexist with the stable laminar state in pipe flows. The boundary between their basins of attraction is the stable manifold of an *edge state*. In pipe flow, this edge state corresponds to a lower branch-traveling wave solution[1].

We demonstrate that a low-dimensional submanifold of this stable manifold, also called the *the edge of chaos*, can be constructed using the recently developed theory of spectral submanifolds (SSMs)[2]. These submanifolds are the unique continuations of spectral subspaces of the linearized system that emanate from stationary states and can be directly constructed from data[3]. Furthermore, in a recent development[4] it was shown that mixed-mode SSMs can also be considered, which are tangent to a subspace spanned by both unstable and stable eigenvectors.

Motivated by the success of spectral submanifold-based reduced-order models for laminar Couette-flows[5], we describe the construction of a two-dimensional, mixed-type spectral submanifold attached to the edge state. The intersection of this manifold with the edge of chaos is a one-dimensional curve, which we can construct using the reduced-order model on the SSM. This yields an explicit parametrization of a low-dimensional footprint of the edge of chaos, visible in the left panel of Fig. 1. The restriction of the flow to this invariant manifold serves as a low-dimensional reduced order model that can capture the transition from the neighborhood of the edge to the laminar state. In the right panel of Fig. 1, we evaluate the performance of the reduced-order model.

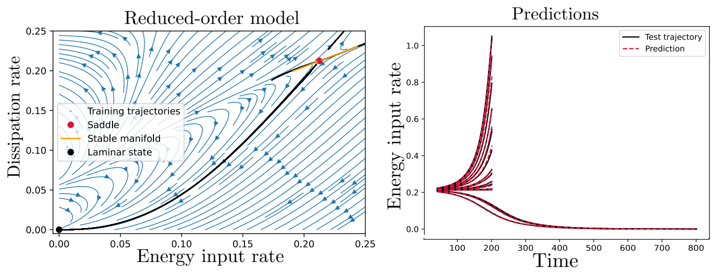

Figure 1: Left: Reduced phase space and the footprint of the edge. Right: Predictions from the reduced-order model.

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[1]Willis et al. *J. Fluid Mech.* **721**, 514 (2013)
[2]Haller and Ponsioen, *Nonlinear Dyn.*, **86**, 1493 (2016)
[3]Cenedese et al. *Nat. Commun*, **13**, 872 (2022)
[4]Haller et al. *submitted* (2023)
[5]Kaszás et al. *Phys. Rev. Fluids*, **7**, L082402 (2022)