Two Fundamental Relations for Turbulent Flows

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This presentation will summarize two exact relations and their use for analysis and modeling of turbulent flows. First, Stokes Flow Regularization (SFR) provides a physics-inspired coarse-graining approach that reveals an exact expression for the energy cascade rate in terms of multiscale velocity gradient dynamics including vortex stretching and strain-rate self-amplification. Beyond single-scale velocity gradient interactions, the role of multiscale interactions of larger-scale strain-rate with smaller-scale vorticity or strain-rate become clear and precise. Applied to direct numerical simulation data, it is revealed that strain self-amplification accounts for the majority of the mean cascade rate. The relative scale-locality of the cascade is inspected in terms of these nonlinear velocity gradient interactions. Calculation of the efficiency of each cascade mechanism reveals that multi-scale interactions tend to behave as an eddy viscosity effect to a much greater extent than the cascade in its entirety. Furthermore, SFR provides a new approach for large-eddy simulation (LES) models. First, SFR produces an alternative to the Germano identity that generates a "pen-and-paper" dynamic procedure for determining model coefficients without the need for a test filter. Unlike traditional spatial filtering theory, SFR leads to governing equations for LES that do not include commutator error terms when the coarse-graining resolution is spatially non-uniform.

A second fundamental relation is the Angular Momentum Integral (AMI) equation for boundary layer flows. Like the so-called FIK relation for the friction factor of internal flows (e.g., channel flow, pipe flow), the AMI equation expresses the skin friction coefficient of boundary layers in terms of the skin friction coefficient of an equivalent laminar boundary layer, plus various contributions to to enhancement or attenuation of skin friction. Physically, the AMI equation represents an integral conservation law for the first moment of momentum deficit, as an extension of von Kármán's momentum integral equation. As such, the AMI equation invites the interpretation of turbulence and freestream pressure gradient effects on boundary layer skin friction in terms of torques that reshape the mean velocity profile in intuitive ways. An analogous relationship for heat flux is easily constructed, and the approach can be applied to both incompressible and compressible flows. For transitional boundary layers, the AMI equation reveals interesting physical effects related to an inversion of the mean wall normal velocity and its effect on the peak skin friction and surface heat flux. The application to high speed boundary layers, up to Mach 7, allows for quantifying the role of density and viscosity variation of skin friction coefficient and Stanton number variation.

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