

# Developing explicit turbulence models by combining gene expression programming and artificial neural network

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Data-driven methods are widely used to develop physical models including turbulence closure models, but there still exist limitations that affect their performance, generalizability and robustness. Compared to deep-learning methods that usually provide “black-box” models, evolutionary algorithms such as gene expression programming<sup>1</sup> (GEP) focus on finding explicit model equations via symbolic regression. However, the random search process in GEP usually slows convergence, and it is difficult to identify accurate model coefficients.

By combining GEP with artificial neural network (ANN), we propose a novel method for symbolic regression called the gene expression neural network (GeNet). In this method, candidate expressions generated by evolutionary algorithms are transformed between the GEP and ANN structures during training iterations, and efficient and robust convergence to accurate models is achieved by combining the GEP’s global searching and the ANN’s gradient optimization capabilities (see Fig. 1). In addition, sparsity-enhancing strategies have been introduced to GeNet to improve the interpretability of the trained models.

The GeNet method has been tested for finding different physical laws, showing improved convergence to models with precise coefficients. Furthermore, for large-eddy simulation of turbulence, the subgrid-scale stress model trained by GeNet significantly improves the prediction of turbulence statistics and flow structures over traditional models, showing advantages compared to the existing GEP and ANN methods in both *a priori* and *a posteriori* tests. We remark that the GeNet method has the potential to be applied to symbolic regression tasks in different fields and applications.

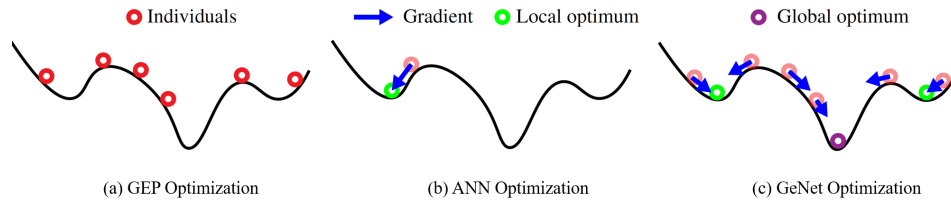


Figure 1: A schematic for the optimization processes of different algorithms.

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<sup>1</sup>Ferreira, *Complex Syst.* **13**, 87–129 (2001).