Anomalous dissipation in turbulence

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The dissipative anomaly, which is sometimes referred to as the zeroth law of turbulence, states that the mean kinetic energy dissipation rate $\langle\epsilon\rangle$ in the bulk of a turbulent flow away from walls remains finite when the kinematic viscosity ν tends to zero. In the present work, we applied the analysis of Duchon and Robert[1] to the turbulent kinetic energy balance of a fully compressible turbulent flow. Next to the anomalous dissipation term D_ε which is given by

$$D_\varepsilon = \frac{1}{4}\int (\nabla\varphi^\varepsilon \cdot \delta\mathbf{u})(\delta\mathbf{u})^2\, d^3\xi, \tag{1}$$

in the incompressible convection case, we obtain additional contributions which arise from the velocity field divergence. Here, $\delta\mathbf{u}$ is the velocity increment in the ξ direction. An anomalous contribution to the energy balances implies that the integral (1) remains finite for the limit of $\varepsilon \to 0$. To calculate these terms in the balances, a continuous test function φ with a compact support is selected using the theory of wavelet transforms. We verified our approach for an analytical test case of an ensemble of Burgers vortices[2]. This approach was suggested more than 20 years ago as a kinematic building block model for the turbulent cascade in three-dimensional homogeneous isotropic turbulence by Hatakeyama and Kambe[3]. The resulting dependence for the dissipation term with respect to ε is displayed in figure 1 (a) for a single vortex and the random ensemble in figure 1 (b). Clearly, D_ε has to vanish in these cases.

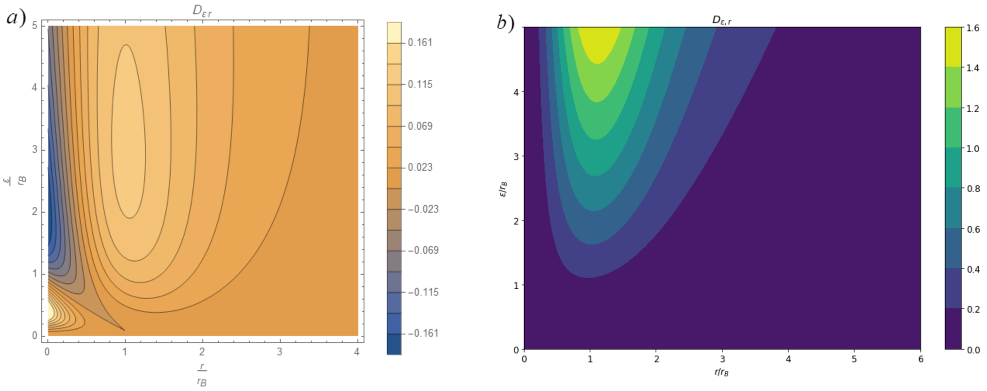

Figure 1: (a) Anomalous dissipation term for a single Burgers vortex for different filter scales ε, (b) same term in an ensemble of randomly strained Burgers vortices.

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[1] Duchon and Robert, *Nonlinearity* **13**, 249 (2000)

[2] Burgers, *J. Adv. Appl. Mech.* **1**, 171 (1948)

[3] Hatakeyama and Kambe, *Phys. Rev. Lett.* **79**, 1257 (1997)