Spanwise vortex structure and orbital instability of unstable periodic orbit in plane Couette turbulence

E. Sasaki[*], G. Kawahara[†] and J. Jiménez[‡]

Dynamical systems theory states that a countably infinite number of unstable periodic orbits (UPOs) are embedded in a chaotic attractor. Through UPO analysis, we could obtain theoretical descriptions for turbulence. However, the high degrees of freedom of developed turbulence make extracting UPOs from turbulence practically hard. We employ the eddy viscosity model in the large-eddy simulation (LES) to decrease the degrees of freedom. If we obtain UPOs embedded in the LES turbulence, their dynamical properties could describe the LES turbulence, from which we could have some perspectives for the NS turbulence. In the LES system, the large scales are computed on the resolved grid points, while the small-scale motions have to be modeled. The models describe the sub-grid-scale stress and the energy transfer between the grid and sub-grid scales. Figure 1-(a) shows a bifurcation diagram of a UPO at $Re = 5000$. The branches can be rescaled using the Smagorinsky mixing length ΔC_S, where C_S is the Smagorinsky constant, the model parameter, and Δ means the sub-grid scale. The bifurcation structure is robust against the resolution, and the eddy viscosity does not violate the phase space structure of the LES system[1]. Figure 1-(b) plots a typical snapshot of UPO at $C_S = 0.400$. We should note that the optimal Smagorinsky constant is $0.04 \sim 0.1$[2]; therefore, this LES UPO is an overdamped solution. Nevertheless, we expect that the LES UPO provides an outlook into the dynamics of NS turbulence, although the eddy viscosity excessively damps the vortices. This LES UPO has spanwise vortices arising from streak instability. These spanwise vortices are stretched by streamwise vortices. The elongated spanwise vortices approach the walls and create new vortices. In the presentation, we will discuss the orbital instability of this UPO using the Lyapunov analysis, which characterizes the tangential space structure and focus on the relationship between the phase-space and vortex structures.

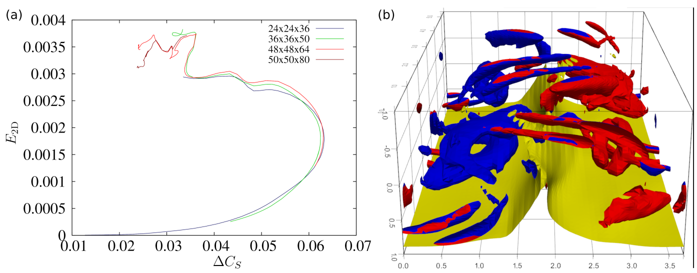

Figure 1: (a) Bifurcation structure of LES UPO at $Re = 5000$ (b) Snapshot of UPO at the Smagorinsky constant $C_S = 0.400$. The red and blue objects mean vortical structure, while yellow one is the isosurface of the streamwise velocity.

---

*Graduate school of Engineering science, Akita University, Akita, Japan.
†Graduate school of Engineering science, Osaka University, Osaka, Japan.
‡School of Aeronautics, Universidad politécnica de Madrid, Madrid, Spain.

[1] Sasaki, Kawahara and Jimenez, *Phys, Rev. Fluid* **6**, 084608 (2021).
[2] Härtel and Kleiser, *J. Fluid Mech.* **356**, 317-357 (1995)