Non-Gaussianity and incompressibility in turbulent relative dispersion

B.J. Devenish[*,†], D.J. Thomson[*]

The problem of turbulent relative dispersion is a classic problem in the study of turbulence and remains a topic of active research. Here we present a Lagrangian stochastic model (LSM) for the dispersion of particle pairs in three-dimensional homogeneous isotropic turbulence (HIT). The LSM is designed to satisfy both the 4/5 law of turbulence and a constraint imposed by incompressibility: in the inertial subrange of turbulence we expect $\overline{u_i a_i} = 3\left(\overline{u_\parallel^3} + \overline{u_\parallel u_p^2}\right)/(2r) = -2\varepsilon$ where $\boldsymbol{a}$ is the relative acceleration, $\boldsymbol{u}$ the relative velocity, $u_\parallel$ and u_p are respectively the longitudinal and absolute transverse components of $\boldsymbol{u}$, r is the absolute separation between the particles, ε is the mean energy dissipation rate and an overbar represents an Eulerian average. Separating the joint distribution of $u_\parallel$ and u_p into its two marginal distributions, as in Devenish & Thomson[1], is not consistent with the incompressibility constraint as then $\overline{u_\parallel u_p^2} = 0$. Instead the constraint can be satisfied by considering the probability density function (pdf) of $u_\parallel$ *conditional* on u_p, $p_{u_\parallel|u_p}(u_\parallel|u_p)$, which we assume to be the product of a polynomial and a Gaussian. We will also need analytical expressions for the first three moments of $u_\parallel$ conditional on u_p and the pdf of u_p, $p_{u_p}(u_p)$. These are based on data from a direct numerical simulation (DNS) of HIT but subject to the expected form of the first three unconditional moments in the inertial subrange i.e. $\overline{u_\parallel} = 0$, $\overline{u_\parallel^2} = C(\varepsilon r)^{2/3}$ (where C is the Kolmogorov constant), $\overline{u_\parallel^3} = (-4/5)\varepsilon r$ and $\overline{u_\parallel u_p^2} = -(8/15)\varepsilon r$. The DNS shows that, on average, smaller values of u_p are associated with small positive values of $u_\parallel$ whereas larger values of u_p are associated with large negative values of $u_\parallel$. In addition, as u_p increases, the variation of $u_\parallel$ about the mean increases and $u_\parallel$ becomes more negatively skewed. Larger values of u_p tend to inhibit the separation of particle pairs. When $p_{u_\parallel|u_p}(u_\parallel|u_p)$ is integrated over $p_{u_p}(u_p)$ we find that the resulting unconditional pdf, $p_{u_\parallel}(u_\parallel)$ agrees well with the DNS data even into the tails of the pdf. The well-mixed condition[2] provides a framework for determining the conditional acceleration given $p_{u_\parallel|u_p}(u_\parallel|u_p)$, $p_{u_p}(u_p)$ and the conditional moments of $u_\parallel$. It will be shown that transverse and longitudinal components of the conditional acceleration agree well with DNS data. A semi-empirical constraint on the well-known non-uniqueness problem[2] will be presented. Richardson's constant, the constant of proportionality in the well known t^3-law for the mean-square separation of an ensemble of particle pairs, will be computed from the LSM for both forwards and backwards dispersion and shown to be consistent with previous results [3] [4] [5] [6].

[*]Met Office, FitzRoy Road, Exeter, UK
[†]Dept of Civil Engineering, Imperial College, London

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