Adjoint-based variational method for constructing invariant solutions of wall-bounded fluid flows

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The dynamics of chaotic fluid flows can be studied in terms of the unstable non-chaotic invariant solutions populating the state space of the governing equations. Invariant solutions such as equilibria, periodic orbits, etc. can be constructed by solving an optimization problem over space-time fields with prescribed temporal behavior: minimization of a cost function that penalizes the deviation of a space-time field from being a solution to the evolution equations. This approach overcomes the sensitivity to the initial guess and small convergence radius, that are fundamental drawbacks associated with the state-of-the-art Newton-based shooting methods.

Despite the advantages of the optimization approach, its application to the 3D wall-bounded fluid flows remains challenging. One challenge is to deal with a very high dimensional problem, and the other is to deal with the non-linear, non-local pressure term which is not easily accessible, if possible at all, in the presence of walls. The adjoint-based minimization techniques introduced by Farazmand[1] and Azimi *et al.*[2] scale linearly with the size of the problem, and address the first challenge. We propose a Jacobian-free algorithm based on these adjoint-based methods for constructing invariant solutions of wall-bounded fluid flows without requiring to construct the pressure field explicitly. We demonstrate the feasibility of the algorithm by constructing equilibria and periodic orbits in plane Couette flow and inclined layer convection (Fig. 1). We also propose a data-driven procedure based on dynamic mode decomposition for accelerating the convergence of the adjoint-descent algorithm.

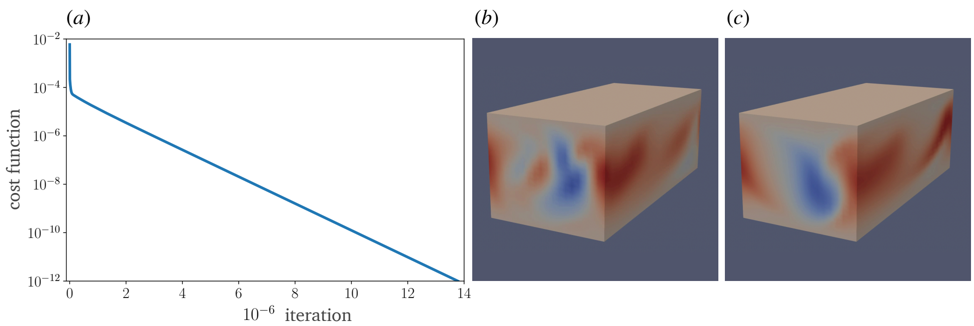

Figure 1: Minimization of the cost function evolves a flow field taken from direct numerical simulation of a chaotic plane Couette flow, panel (b), towards a true equilibrium solution, panel (c). Contour plots display the wall-normal component of the velocity.

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[1] Farazmand, *J. Fluid Mech.* **795**, 278–312 (2016).

[2] Azimi, Ashtari and Schneider, *Phys. Rev. E* **105**, 014217 (2022).