Turbulent Circulation Statistics on Non-Planar Contours

L. Moriconi[*], R.M. Pereira[†], and V.J. Valadão[*]

Turbulent fluctuations of circulation have peculiar statistical properties which have come to light in recent years as the result of massive direct numerical simulations.[1,2] These studies revive in a vigorous way the long-standing idea that circulation is a key observable for a deeper theoretical understanding of important issues in the statistical theory of turbulence.[3,4] We mean here the existence of a stable energy cascade in the asymptotic limit of high Reynolds numbers, and the connection between the statistical description of vortex structures and multifractal inertial range scaling.

Some of the challenging phenomenological features of turbulent circulation on planar contours, such as the shape of probability distribution functions (PDFs), the scale-dependent behavior of the circulation flatness, and the scaling exponents of circulation moments, have been accurately addressed along the lines of a vortex gas model of homogeneous and isotropic turbulence.[5,6,7,8] This modeling framework combines the structural view of turbulence, taken as a system of diluted vortex tubes, with the Gaussian multiplicative chaos (GMC) formulation of the energy cascade[9] (a field theoretical generalization of the well-known Obukhov-Kolmogorov theory of intermittency.[10,11])

We report, in this work, possible extensions of the vortex gas model to the case of non-planar circulation contours. Previous theoretical and numerical investigations suggest that the minimal surfaces bounded by such contours determine the shape of properly rescaled circulation PDFs.[2,3] Unbiased by these indications, we look for the bounded surfaces which lead to consistent vortex gas model evaluations of the circulation PDFs. Our computational strategy is implemented along two not necessarily equivalent ways, through the realization of (i) GMC in three-dimensional space and (ii) a variation of the two-dimensional GMC model which is suitable for application on curved surfaces. We are able, in this way, to critically revisit the role (and mathematical necessity) of minimal surfaces within the context of the vortex gas model of circulation statistics.

---

[*]Instituto de Física, Universidade Federal do Rio de Janeiro, C.P. 68528, CEP: 21945-970, Rio de Janeiro, RJ, Brazil

[†]Instituto de Física, Universidade Federal Fluminense, CEP: 24210-346, Niterói, RJ, Brazil

[1]K.P. Iyer, K.R. Sreenivasan, and P.K. Yeung, *Phys. Rev. X* **9**, 041006 (2019).

[2]K.P. Iyer, S.S. Bharadwaj, and K.R. Sreenivasan, *Proc. Natl. Acad. Sci. U.S.A.* **118**, e2114679118 (2021).

[3]A.A. Migdal, *Loop equation and area law in turbulence* in *Quantum Field Theory and String Theory*, edited by L. Baulieu, V. Dotsenko, V. Kazakov, and P. Windey, Eds. (NATO ASI Series), Springer, Boston, MA, vol. 328, 193 (1995). See also A. Migdal, *Phys. Rep.* **1011**, 1 (2023).

[4]L. Moriconi, *Proc. Natl. Acad. Sci. U.S.A.* **118**, e2117968118 (2021).

[5]G.B. Apolinário, L. Moriconi, R.M. Pereira, and V.J. Valadão, *Phys. Rev. E* **102**, 041102 (2020).

[6]L. Moriconi, *Phys. Rev. E* **103**, 062137 (2021).

[7]L. Moriconi, R.M. Pereira, and V.J. Valadão, *Phys. Rev. E* **106**, L023101 (2022).

[8]L. Moriconi and R.M. Pereira, *Phys. Rev. E* **106**, 054121 (2022).

[9]R. Robert and V. Vargas, *Ann. Probab.* **38**, 605 (2010).

[10]A.M. Obukhov, *J. Fluid Mech.* **13**, 77 (1962)

[11]A.N. Kolmogorov, *J. Fluid Mech.* **13**, 82 (1962).