Heat flux and wall shear stress in turbulent natural convection in a differentially heated vertical channel

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Turbulent natural convection in a differentially heated vertical channel is an idealized model system for studying thermally-driven wall-bounded turbulent flows. We have carried out a theoretical study in the large aspect-ratio limit, where the height and width of the vertical walls are much larger than their separation. Our theoretical analysis yields two relationships between heat flux and wall shear stress, measured respectively by the Nusselt number (Nu) and shear Reynolds number (Re_τ), and their dependence on the Rayleigh (Ra) and Prandtl numbers (Pr) in the high-Ra limit[1]:

$$\begin{aligned} \mathrm{Nu} &\approx [C^2 f(\mathrm{Pr})]^{1/3}\mathrm{Pr}^{-(1-2\varepsilon)/3}\mathrm{Ra}^{1/3} \\ \mathrm{Re}_\tau &\approx [f(\mathrm{Pr})/C]^{1/3}\mathrm{Pr}^{-(1+\varepsilon)/3}\mathrm{Ra}^{1/3} \end{aligned}$$

Here, C is a constant, $f(\mathrm{Pr})$ is a function of Pr that is not in the form of a power law and $\varepsilon = 1/3$ and 1, respectively for $\mathrm{Pr} \gg 1$ and $\mathrm{Pr} \ll 1$. These theoretical results imply that data points of Nu and Re_τ taken at different values of Pr can be collapsed into single curves of $\mathrm{Ra}^{1/3}$-dependence for large Ra, when multiplied by appropriate factors of $f(\mathrm{Pr})$ and Pr. As shown in Fig. 1, this is confirmed by direct numerical simulation (DNS) data[2] for $\mathrm{Pr} \geq 1$ with the values of $f(\mathrm{Pr})/f_0$, where $f_0 \equiv f(\mathrm{Pr} = 10)$, estimated from the DNS data.

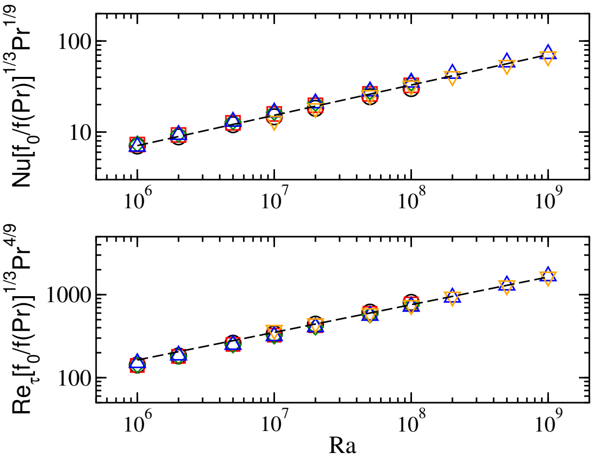

Figure 1: Collapse of DNS data for Nu and Re_τ at different values of Pr [Pr = 1 (circles), 2 (squares), 5 (diamonds), 10 (triangles) and 100 (inverted triangles)] in agreement with the theoretical results. The dashed lines are the best fits for the theoretical prediction of $\mathrm{Ra}^{1/3}$-dependence.

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[1] E.S.C Ching, *Phys. Rev. Fluids*, **8**, L022601 (2023).

[2] C.W. Howland, C.S. Ng, R. Verzicco and D. Lohse, *J. Fluid Mech.*, **930**, A32 (2022).