Machine-aided initial guesses for unstable periodic orbits

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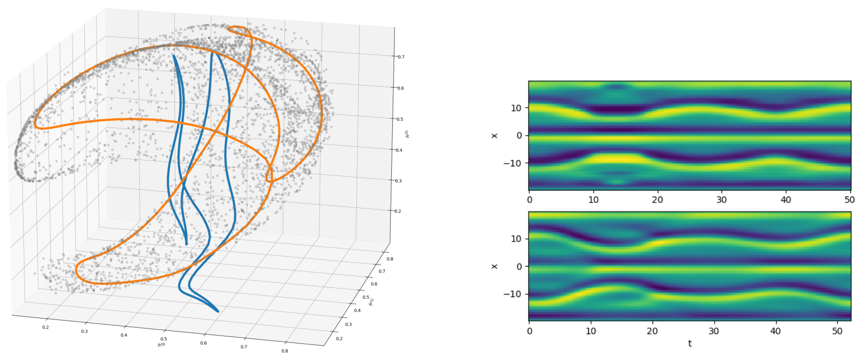
Unstable periodic orbits (UPOs) are believed to be the underlying dynamical structures of turbulence. Finding these UPOs is notoriously difficult however. Conventionally, shooting methods were the method of choice, where an initial condition is optimised until it is revisited by its forward trajectory. While they in theory work in high-dimensions, exponential error amplification inherent to chaotic systems causes small convergence radii, rendering the approach ineffective. A recent alternative are matrix-free[1] loop convergence algorithms[2], in which entire space-time fields (loops) are deformed until they satisfy the evolution equations. Initial guesses for the variational convergence algorithms are thus space-time fields in a high-dimensional space rendering their identification highly challenging.

While the dimension of the space used to discretize fluid flows is prohibitively large to construct suitable initial guesses, the dissipative dynamics will collapse onto a chaotic attractor embedded in a curved manifold of far lower dimension. We use data-driven methods to construct an approximation of the coordinates for this manifold, and leverage this reduction in dimensionality to construct initial guesses for loops. More specifically, we use convolutional autoencoders to obtain a low-dimensional representation of the discretized physical space. In the low-dimensional latent space, we define ad-hoc loops, which are decoded to physical space and used as initial guesses. Together with variational convergence algorithms, we quickly converge to UPOs (see figure 1). We illustrate our methods in the 1D Kuramoto-Sivashinsky system, which exhibits turbulence at easily computable parameter ranges.



(a) (b)

Figure 1: The discretized spatio-temporal complex KSEq (b) is reduced to a 3-dimensional system (a), with the attractor represented by grey dots. An ad-hoc guess (blue in (a), top in (b)) is converged to an UPO (orange in (a), bottom in (b)).

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[1]Azimi et al., *Phys. Rev. E*, **105**, 014217 (2022).

[2]Lan & Cvitanović, *Phys. Rev. E*, **69**, 016217 (2004).