Ghost hunting: finding solutions which don't exist

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It has long been known[1] that close to saddle-node bifurcations, the behaviour of a system can closely match that of the bifurcating invariant solution even when the solution is not present at the chosen parameter value. Until now, no computational method has been able to find and continue such 'ghost' solutions. Thanks to recently developed algorithms[2,3], this is now possible, as ghosts manifest as local minima in an optimization problem with a carefully constructed objective function. This is tractable for steady state solutions and periodic orbits, and even for more exotic structures such as (non-)invariant tori.

The family of methods is demonstrated for simple toy dynamical systems, including a 1D map, the Lorenz ODE system and turbulence in the Kuramoto-Sivashinsky PDE, where ghosts are converged and continued, and used to explain the observed chaotic behaviour. We then present results of applying our methods to weakly turbulent two- and three-dimensional Navier-Stokes equations, and in particular we use ghosts to explain a previously observed phenomenon in inclined layer convection[4] (see figure 1).

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[2] Azimi et al., *Phys. Rev. E* **105**, 014217 (2022).
[3] Parker & Schneider, *J. Fluid Mech.* **914**, A17 (2022).
[4] Reetz et al., *J. Fluid Mech.* **898**, A23 (2020).

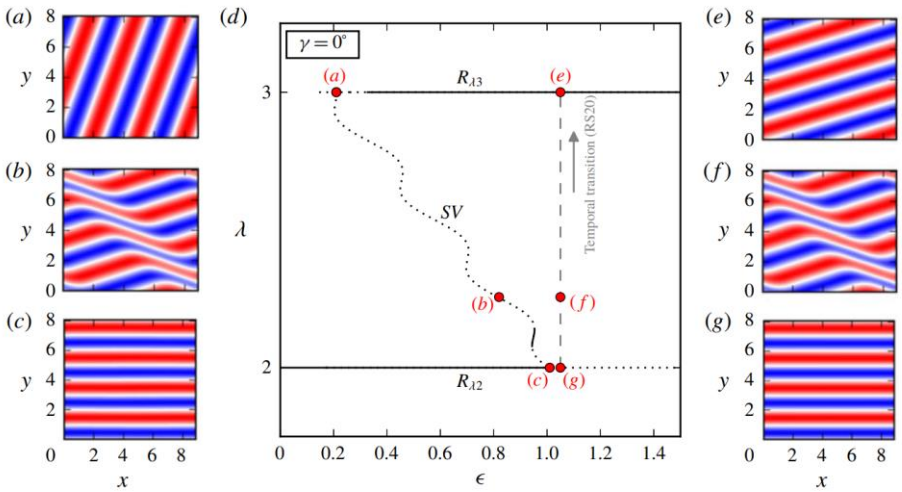

Figure 1: The transition (f) between spatial patterns in inclined layer convection nearly exactly matches the 'skewed-varicose' invariant solution (b), despite this solution not existing at the studied parameters. This ghost has now been computationally found and continued. Figure taken from ref. 4.