Diagnostic of Fourier triad-phase synchronisation via a phase-frozen model in 1D Burgers and 3D Navier-Stokes flows

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Nonlinear triadic interaction plays a key role in the multiscale energy transfer of turbulence. It has been observed that the Fourier triad phases are synchronised to $\frac{\pi}{2}$ when energy transfer is enhanced in the 1D Burgers flow, either with the stochastic forcing[1] or in the developing process from specific initial conditions[2]. The focus of this work is twofold. The first objective is to understand what triggers phase synchronisation. We revisit the 1D Burgers equation with a staircase initial condition[3]. It is found that all the triad phases are strictly synchronised to $\frac{\pi}{2}$ as the energy spectrum develops. A recursive formula of triad-phase relation is proposed to demonstrate that this strict synchronisation is triggered by the convective term in the Fourier space. On the contrary, 1D Burgers developing from a continuous energy spectrum exhibits a wide-range triad-phase distribution maximised at $\frac{\pi}{2}$, of which the shape depends on the scaling law of the initial energy spectrum. The second objective is to develop a simple diagnostic tool to examine when and where the phase synchronisation occurs, which is adapted not only for 1D Burgers flow but also for 3D Navier-Stokes turbulence. We propose a frozen-phase model in the sense that only the Fourier amplitudes evolve with time, with a fixed set of Fourier phases prescribed. By freezing the phases at large and small scales, respectively, and inspecting the temporal evolution of dissipation rates, it is clearly shown that the triad-phase synchronisation occurs at small scales, consistent with the spectral triad-phase distributions in Fig. 1. It is also found that the phase synchronisation takes place in a time scale much shorter than that for the scaling law k^{-2} to be established in the inertial scale. By applying the frozen-phase model in the diagnostic of 3D isotropic turbulence, it is found that the turbulence statistics evolve slower with time, but the general tendency is not significantly affected by freezing phases.

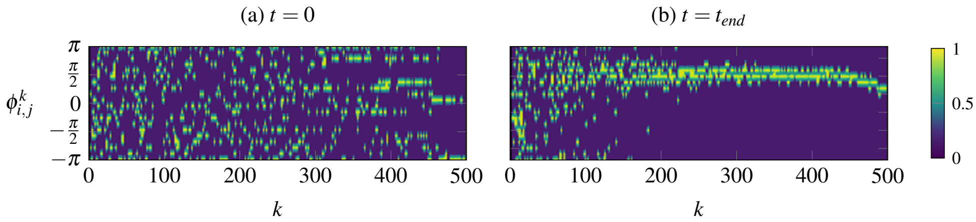

Figure 1: Spectral distribution of triad phases $\phi_{i,j}^k$ where the triad wavenumbers satisfy the relation $i+j=k$.

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[2] Kang et al., arXiv:2105.09425
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