Linear and nonlinear instability of supersonic boundary layers over a rotating cone

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Laminar-turbulent transition in supersonic boundary layers is of practical importance due to its direct relevance to aerodymamic designs in high-speed flying vehicles. In this talk, we particularly focus on the effect of the rotating rate on the instability of a sharp-cone boundary layer as sketched in figure 1-(a), which is a typical three-dimensional boundary layer and may be related to different linear and nonlinear instability regimes. Setting the oncoming Mach number to be 3 and the semi-apex angle to be 7 degree, we calculate the base flow by solving the compressible boundary-layer equations using a marching scheme. Such a calculation is confirmed to be accurate by comparing with the full Navier-Stokes solutions. Linear stability analysis reveals that there exist three types of modes, i.e., the modified Mack mode (**MMM**), cross-flow mode (**CFM**) and the centrifugal mode (**CM**), and preliminary instability characteristics are reported in Ref [1].

To consider the nonlinear interaction between different Fourier components, we introduce a pair of oblique instability modes at the inlet of a selected computational domain and solve the nonlinear parabolized stability equation (NPSE) numerically. As the rotating rate increases, three distinguished nonlinear regimes, i.e., the oblique-mode breakdown (**OB**), the generalized fundamental resonance (**GRF**) and the centrifugal-instability induced transition (**CIT**), appear in sequence, and their flow patterns are shown in figure 1-(b). For each regime, the mechanisms for the amplifications of the streak mode and the harmonic travelling waves are explained in detail, and the dominant role of the streak mode in triggering the breakdown of the laminar flow is particularly highlighted. Additionally, it is found that the transition thresholds should vary with the nonlinear-resonance regimes, and the traditional transition prediction method, the e-N method, based on the linear instability should be carefully employed if multiple nonlinear regimes may appear.

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[1] Song and Dong, *J. Fluid Mech.* **955**, A31 (2023).

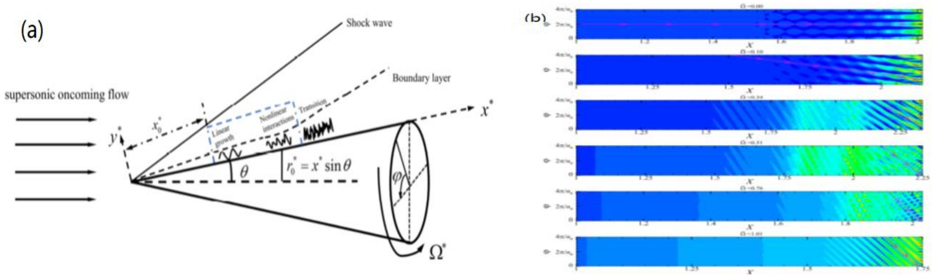

Figure 1: (a) Sketch of the physical mode. (b) Different transition regimes.