Efficiency of Turbulence and weak dissipative solutions of Navier-Stokes

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In fluids, mechanical energy can be converted into thermal energy via two-steps: first, the creation of finer and finer energy-transporting structures (eddies); then, as the size of the structures reach the Kolmogorov length η, structures breaking and transformation into heat. In the process, some finite amount of mechanical energy is stored within the fluid. The efficiency of this storage can be measured by the non-dimensional number $\mathcal{E} = E/E_i$, where E is the stored kinetic energy per unit mass and E_i is a measure of the mechanical injected energy input per unit mass. Despite its obvious interpretation, this number is seldom used in turbulence. However, the independence of this number with respect to the viscosity makes this number suitable for exploring universal asymptotic regimes of turbulence, where viscosity does not play a role anymore. In this talk, we will show an example of application of this idea to the problem of injection and dissipation of energy within a flow, and its connection to potential singular solutions of the incompressible Navier-Stokes equations.

We have explored properties of numerical Navier-Stokes solutions under two protocoles, one at fixed viscosity, variable energy ("canonical") and one at fixed energy, variable viscosity ("microcanonical"). This second protocole allows us to explore a region parameter space that is not accessible by the traditional procedure. In this region, at finite resolution, we find solutions that are mixture of scale invariant solutions at small wavenumber, and thermalized solution at high wavenumber see Fig. 1. In the limit of infinite resolution, we conjecture that these solutions converge to dissipative weak solutions of Euler equation, foreseen by Onsager [1]. Current construction of such solutions is usually done via convex integration methods, which are delicate to implement. If the conjecture holds true, it means that the microcanonical protocol could provide a new way of constructing those solutions through a controlled limit procedure. As such, it would probably pave the way to new exact results regarding the existence and properties of dissipative weak solutions of Euler.

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[1]Onsager, Lars, Il Nuovo Cimento, 6, 279–287, (1949)