Composition of resolvents enhanced by random sweeping for large-scale structures in turbulent channel flows

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Resolvent analysis has been developed mainly for spatial structures but rarely used for temporal statistics[1], since white-in-time models for nonlinear forcing lead to incorrect correlations in time[2,3]. In this paper, composite resolvents[4] are introduced to reproduce frequency spectra of large-scale structures in turbulent flows. The new developments are as follows: (1) composition of resolvents is proposed to avoid the direct action of white-in-time nonlinear forcing on velocity fields; (2) the random-sweeping effect is introduced to represent time decorrelations. The numerical results for turbulent channel flows are used to verify the composite sweeping-enhanced resolvents.

According to resolvent analysis, the Navier–Stokes equations are written as $\tilde{\boldsymbol{u}} = \boldsymbol{R}\tilde{\boldsymbol{F}}$, where $\boldsymbol{u}$ is the velocity fluctuation, $\boldsymbol{R}$ is the resolvent, $\boldsymbol{F}$ is nonlinear forcing, and $\tilde{\cdot}$ denotes the space–time Fourier mode. The random sweeping effect of nonlinear forcing is represented by eddy damping, leading to a sweeping-enhanced resolvent $\boldsymbol{R}_s$. Furthermore, the remaining effects of nonlinear forcing, $\tilde{\boldsymbol{f}}$, is modelled as the output of $\boldsymbol{R}_s$ from white-in-time input $\tilde{\boldsymbol{f}}_t$, that is, $\tilde{\boldsymbol{f}} = \boldsymbol{R}_s\tilde{\boldsymbol{f}}_t$. Therefore, the velocity fluctuation $\tilde{\boldsymbol{u}}$ is determined by the random forcing $\tilde{\boldsymbol{f}}_t$ through the composition of the sweeping-enhanced resolvent $\boldsymbol{R}_s$ such that $\tilde{\boldsymbol{u}} = \boldsymbol{R}_s^2\tilde{\boldsymbol{f}}_t$. The resulting $\boldsymbol{R}_s^2$ model correctly yields the frequency spectra (see Fig. 1), two-point cross-spectra and the characteristic decay timescales of large-scale structures in turbulent channel flows.

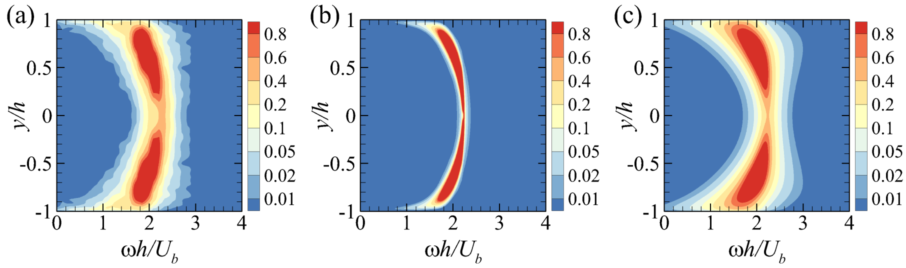

Figure 1: Frequency spectral distributions in the wall-normal direction, $\Phi(\boldsymbol{k},\omega,y)/\Phi_I(\boldsymbol{k})$, at $\boldsymbol{k}h = (2,4)$ for large-scale structures at $\mathrm{Re}_\tau = 550$. (a) DNS. (b) ν model. (c) $\boldsymbol{R}_s^2$ model.

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